

## ON THE ROLE OF DEFICIT ZONES OR IMPERFECTIONS IN DRY SNOW SLAB AVALANCHE RELEASE

Jürg Schweizer\*

Swiss Federal Institute for Snow and Avalanche Research, Davos, Switzerland

**ABSTRACT:** Dry snow slab avalanche formation starts with failure in the weak layer underlying the slab. The stress has to locally exceed the strength that strongly depends on the strain rate. Providing the deformation energy released is sufficient, a shear fracture might propagate and eventually slab release takes place. Whereas field measurements suggest that skier triggering is common without hitting a deficit zone, it seems plausible that in the case of natural release only with the presence of imperfections the critical strain rate and strain can be reached to initiate the brittle fracture in the slab failure process. However, little is known about the type and size of these imperfections, and in particular about their lifespan. Model calculations give a wide range of values on the size, typically between 0.1 to 10 m. It seems unlikely that a deficit zone will survive for weeks (and wait for a skier), but rather will heal pretty soon within hours. Deficit zones are supposed to be a transient phenomenon. This might partly explain why there is hardly any direct evidence for imperfections from field studies e.g. on the spatial variability of the snow cover stability. Both, natural release and artificial triggering by skiers of dry snow slab avalanches are revisited.

**KEYWORDS:** snow mechanics, avalanche mechanics, avalanche release

### 1. INTRODUCTION

It seems generally clear and accepted that dry snow slab avalanche release starts with shear failure in a thin weak layer (or at an interface) underlying a relatively thick cohesive slab (McClung and Schaerer, 1993). Whereas by far most avalanches release naturally during or shortly after storms (direct-action avalanches), the majority of avalanche victims in the developed countries are skiers that triggered the fatal avalanche themselves. Laboratory experiments have clearly shown that snow strength is in general rate dependent (e.g. Narita, 1983). To induce failure the shear stress by the slab has to locally exceed the shear strength in the weak layer, so that the rate of deformation and the deformation increases. It is obvious that by artificial triggering (e.g. by explosives or skiers) the critical rate might be attained. However, for naturally released avalanches it seems more difficult to fulfill the rate condition. Most present models of dry snow slab avalanche release (McClung, 1979, 1981, 1987; Bader and Salm, 1990) are based on principles from linear fracture mechanics. They assume that the stress relaxation at the borders of an imperfection within the weak

layer where the stress is concentrated should provide the energy needed for the formation of a new free surface (crack) and so enable fracture propagation. Based on an energy balance, the size of the imperfection for failure/fracture propagation can be assessed. In the models the imperfections are defined as zones of zero or residual strength so that the shear stress due to the overlaying layer, the slab, cannot be supported.

The idea of having weaker parts within the weak layer (in the neutral zone) is quite old and goes back to Haefeli (1967): "... there occurs a kind of regression of the inner friction in the slope-parallel potential sliding layer...". These weaker zones within the weak layer have been called: shear perturbations (Perla and LaChapelle, 1970), shear degeneration (Brown, Evans and LaChapelle, 1972), imperfections (Lang and Brown, 1975), zonal weakening of the snowpack (Bradley et al., 1977), deficit zones (Conway and Abrahamson, 1984), shear bands or slip surfaces (McClung, 1979, 1981, 1987), superweak zones (Bader and Salm, 1990), tender spots (Logan, 1993), zones of localized weakness (Birkeland et al., 1995), or in general flaws or weak spots. However, the last term, weak spot (and also tender spot), is usually used to characterize a weak part in the snow cover. It is rather used for the case of a locally different, weaker snow structure (different layering), not for a weaker part within a weak layer. Only the latter type will be considered in the following. Whereas it is quite obvious how weak spots

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\* Jürg Schweizer, Swiss Federal Institute for Snow and Avalanche Research, Flüelastrasse 11, CH-7260 Davos Dorf, Switzerland; tel: +41 81 417 0164; fax: +41 81 417 0110; email: schweizer@slf.ch

develop (e.g. around rocks, or in general where the snow depth is reduced, and consequently the temperature gradient is increased), not much is known where deficit zones come from. In the following imperfections, deficit zones and superweak zones will be used synonymously.

Imperfections and in particular their critical size, are assumed to be a key parameter to be considered for stability evaluation and hence avalanche forecasting. The size is supposed to be of the order of 5 to 10 times the slab thickness (Gubler, 1992). However, as Salm (1982) points out: "Unfortunately, in a snow cover no obvious formation process is available for such weakness." It is not clear how to proceed from bond fractures to slab release. Although, there are some plausible explanations since then (Gubler and Bader, 1989). Nevertheless, it seems that the term has become pretty nebulous. Any rather surprising avalanche release is attributed to a superweak zone. Furthermore the use of stability tests has been generally questioned (Salm, 1986). The crux seems to be that the models cannot be verified, but rather present plausible explanations. In the following an attempt is made to review the role of superweak zones in avalanche release. One of the main questions is: How is a superweak zone born, how is its lifespan, and how does it disappear?

## 2. SNOW FAILURE

Laboratory experiments have shown that the mechanical behavior of snow depends on the rate of loading (for shear loading see McClung, 1977; deMontmollin, 1982; Fukuzawa and Narita, 1993; Schweizer, 1998). At low rates snow shows predominately non-linear viscous behavior. Considerable strain energy can be dissipated. At high rates the elastic properties dominate and samples break after very limited deformation (brittle failure). Schweizer (1998) found the transition between the ductile and the brittle state of failure was at a strain rate of about  $1 \times 10^{-3} \text{ s}^{-1}$ , for the snow type tested (small rounded particles, size  $\leq 0.5 \text{ mm}$ ). For larger grains (size  $\approx 1 \text{ mm}$ ) the transition is shifted towards lower strain rates at about  $1 \times 10^{-4} \text{ s}^{-1}$ . For well consolidated snow (density  $> 150 \text{ kgm}^{-3}$ , in-situ hardness about four fingers and more), samples dilate during shearing and show strain softening. Strain softening means, interpreting a classical stress-strain curve, that the stress decreases after a certain amount of displacement, either until fracture occurs, or the stress reaches eventually a residual stress. This behavior is due to changes at the micro-scale (scale of bonds). For the case of

concrete the existence of strain softening is interpreted as fracture, cracking or other damage implying that the failure process would be non-simultaneous and propagating (Bazant, 1992, p.8). In general, for viscoelastic materials brittle and ductile failure behavior can be related to the strain. Brittle failure is attributed to volumetric strain and ductile failure to distortion strain. Snow has as most materials, the ability to absorb both volumetric and distortion strain energy. The magnitude and relation between these two energy forms are dependent on the loading rate (Sentler, 1987).

As for any viscoelastic material, the snow strength also depends on strain history (Brown et al., 1973). Sentler (1987) describes viscoelastic behavior in general as to be due to the existence of microflaws, inherently present in such materials. Already small deformations cause damage, and damage accumulation might eventually lead to failure. There is only initially a very small linear viscoelastic range where no damage occurs. Camponovo and Schweizer (1997b) found that the behavior is linear viscoelastic (i.e. no damage occurs, repeated loading gives an identical response) below a shear strain of only about  $1 \times 10^{-4}$ , independent of rate. So virtually any deformation implies damage or structural changes (not including the processes of snow metamorphism).

Damage and in particular damage accumulation can be related to acoustic emission. Rapid release of strain energy from localized sources within a body, such as regions of stress concentration, leads to micro-seismic activities and the emission of acoustic waves (Sinha, 1996). Emissions are in the ultrasonic (kHz) range.

At the micro-scale, deformation and failure of snow can hence be understood in terms of two competing effects: damage (e.g. fracturing of bonds) and formation of bonds (healing or sintering). Sintering might play a key role and has not been considered in most mechanical models. The effect of sintering in shear deformation must decrease as deformation rates increase, and as snow temperature decreases. However, deMontmollin (1982) proposes that even during fast loading after brittle fracture within a very short period of time some considerable strength is building up. This leads to a zigzag stress-strain curve. Traditionally sintering (or growth of bonds) has first of all been considered as a slow process: bond growth primarily due to the transport of water molecules through the vapour phase (e.g. Ramseier and Sander, 1966). But under loading conditions initial sintering can happen within seconds. Accordingly, Gubler (1982) designed an experiment to study the strength of bonds between ice grains after short

contact times. The results support the idea that fast sintering gives some initial strength.

In other materials as concrete pores are considered as points of weakness, as microflaws, where stress is concentrated. As snow is a highly porous medium, stress concentrations and damage at the micro-scale is likely to be abundant. However, it is not clear whether coalescence of diffuse microcracks leads to fracture localization, also called fracture nucleation (i.e. a crack can be identified) that could be a starting point at the macroscopic scale for fracture propagation. Narita (1983) has found microcracks during tensile tests after substantial deformation (0.13) at slow rates (between  $10^{-6}$  and  $10^{-5}$  s $^{-1}$ ), but McClung (personal communication) could never find any during his shear tests. Narita (1983) also found small cracks adjacent to a tensile crack (of ductile type) in the snow cover.

However, fracture mechanics of other materials as concrete (or disordered material in general) strongly suggests that localized cracking follows from diffuse microcracking (damage). Since microflaws in snow must abundantly exist (see argument above), one could conclude that this would lead to constant avalanching. However, the same fundamental fluctuation (or disorder) does not only provide points of failure, but as well points of higher strength that represent a necessary stabilizing mechanism to prevent the localization of damage that would lead to catastrophic fracture (Herrmann and Roux, 1990). Thus a statistical approach would not only explain failure initiation in weak zones but also fracture arrest or healing (sintering) in stronger zones, so that the number of avalanches would become limited again.

Sommerfeld (1973) first proposed the use of

statistical models to describe snow strength. The force transmitting elements, links or chains can be arranged in series or parallel. Series and parallel approach can be described with the models of Weibull and Daniels, respectively. For high strain rates with elastic behaviour causing brittle fracture, Weibull's theory seems appropriate, whereas for ductile failure Daniels' theory should be applied. Gubler (1978a,b) followed the statistical approach using quantitative stereology and established strength-structure relations based on a description of chains. Although snow can not always be characterized as chains, the statistical approach seems in general highly appropriate for snow, considered as a disordered material. But hardly any work has been done during the last ten to twenty years.

### 3. NUMERICAL VALUES FOR SNOW SLAB FAILURE MODELS

Based on the review by Mellor (1975) and more recent field measurements (Jamieson and Johnston, 1990; Jamieson, 1995; Föhn, 1993) and laboratory studies (McClung, 1977; Schweizer, 1998; Camponovo, 1998: unpublished research report) a set of mechanical parameters was chosen and used (Table 1) to assess and compare the results of the snow slab failure models presented in the following section. The values characterize the weak layer and the overlying slab. A range is given that shows typical variation (not to be considered as minimal and maximal values). Field data of strength usually represents the brittle range, laboratory data covers strain rates between  $5 \times 10^{-6}$  s $^{-1}$  to  $5 \times 10^{-3}$  s $^{-1}$ , and is mostly for snow consisting of small rounded particles.

Table 1. Typical values for slab and weak layer

Parameter	typical value	range
Density: $\rho$	200 kg m $^{-3}$	100 - 300 kg m $^{-3}$
Slope angle: $\psi$	38°	30° - 45°
Poisson's ratio: $\nu$	0.2	0.1 - 0.4
Weak layer thickness: $d$	10 mm	1 - 15 mm
Slab depth (perpendicular): $H$	0.5 m	0.3 - 1 m
Tensile strength/weight: $\sigma_t/\rho g$	1 m	0.5 - 3 m
Young's modulus: $E$	1 MPa	0.5 - 10 MPa
Residual strength/peak strength: $\tau_r/\tau_p$	1.5	1.25 - 2.0
Shear modulus: $G$	0.5 MPa	0.1 - 5 MPa
Shear viscosity of slab: $\eta_0$	$5 \times 10^8$ Pa s	0.5 - $10 \times 10^8$ Pa s
Shear viscosity of weak layer: $\eta_s$	$0.5 \times 10^8$ Pa s	0.1 - $1.5 \times 10^8$ Pa s
Modulus to peak stress (fast): $G/\tau_p$	500	300 - 1000
Modulus to peak stress (slow): $G/\tau_p$	100	20 - 200

In the following for all models a coordinate system with origin at the snow surface is used with the x-axis pointing downslope and the z-axis pointing downward into the snow cover. The y-axis would point along a contour line, but only 2D models are considered in the following. Accordingly  $\tau_{xz}$  denotes the shear stress, and  $\sigma_{zz}$  the normal stress. The shear stress due to the slab is given as  $\tau_g = \rho g H \sin \psi$ , with  $\rho$  average slab density,  $g$  acceleration due to gravity,  $H$  slab thickness (measured slope perpendicular) and  $\psi$  slope angle. Other parameters used are either given in Table 1 or subsequently defined in the text.

#### 4. SNOW SLAB FAILURE MODELS

In this section some of the snow slab failure models for natural release that include a size effect, e.g. a critical length for fracture (propagation), are briefly reviewed, compared and evaluated with the help of the above defined mechanical parameters. These models were often motivated by lack of explanation for delayed action avalanches. It might be helpful to point out that until about 1970 several cases have been considered for primary fracture. Beside compressive fracture at the base of the slab, shear fracture in the weak layer and tensile fracture of the slab have been heatedly discussed. Perla and LaChapelle (1970) who assumed that first fracture was in tension at the crown, have given paradoxically the best argument for primary shear fracture which is nowadays commonly accepted. Although tensile crown fracture could never be completely ruled out, and the collapse of a weak layer (compressive failure) seems in fact quite plausible as well. The question would be whether ductile failure only in the weak layer (e.g. a decrease of strength to a residual value of 50%) would be sufficient to cause tensile fracture in the slab. Considering that shear and tensile strength are not so much different, the case might not be so clear. Of course, still, the initial failure would be in shear.

One of the first that considered a shear perturbation was Jaccard (1966). He studied the effect of a local fracture in the weak layer, in particular the stress concentration at the edges of the local fracture. The result is that the stress concentration is about 4 times the applied shear stress due to the slab, assuming that the Poisson's ratio is about 0.2. Jaccard's solution is not very realistic as it is independent of the length of the imperfection, following from his assumption that the stress

perturbation is inversely proportional to the distance from the center of the imperfection. He also considered the effect of an additional load by a skier. However, his work (in French) received no further attention.

Perla and LaChapelle (1970) determined the order of magnitude of the length of a shear perturbation with the view to explain slab release by the resulting tensile stress concentration. By scale analysis of the equilibrium equations a shear perturbation of the order of  $O(\tau'_{xz})$  would increase the tensile stress by the order of  $O([L/H] \tau'_{xz})$ , where  $2L$  is the length of the perturbation and  $H$  is the slab thickness. Introducing as local shear strength, the residual shear strength  $\tau_r$ , the relation turns into:

$$\sigma_{xx} \approx (L / H)(\tau_g - \tau_r) \quad (1)$$

The expression  $(\tau_g - \tau_r)$  can be called shear deficit. Perla (1980) shows that a shear loss of 50% over e.g. a distance of ten times the slab thickness would increase the tensile stress by factor 5 and would rotate the direction of the principal stress (in tension) to within  $10^\circ$  of slope perpendicular, consistent with the observed crown-bed surface fracture angle. Perla concludes that a reduction in shear support (not a fracture) possibly caused by strain softening leads to tensile fracture at the crown which is followed by, or synchronised with, basal shear fracture.

Brown, Evans and LaChapelle (1972) have calculated (following Perla and LaChapelle, 1970) in more detail the additional stress induced by the reduction of the shear strength over some area of the interface. The analysis is carried out by assuming a reduction to zero of the shear strength over a metamorphosed basal rectangular region. They receive for the dimensional ratio

$$\frac{L}{H} = \frac{\sigma_t}{\tau_{xz}} + \frac{\nu}{2(1-\nu)} \cot \psi \quad \text{or}$$

$$L = \frac{\sigma_t}{\rho g \sin \psi} + \frac{\nu}{2(1-\nu)} H \tan(90^\circ - \psi) \quad (2)$$

with  $\sigma_t$  the tensile strength.

Following the model of Perla and LaChapelle (1970), Lang and Brown (1975) calculated the state of stress in the vicinity of a basal layer shear imperfection. They find large stress concentration close to the discontinuity. Curtis and Smith (1974) and Smith and Curtis (1975) also studied the stress concentration at the edge of the weakened sublayer applying the finite element method, and confirmed Perla and LaChapelle's results.

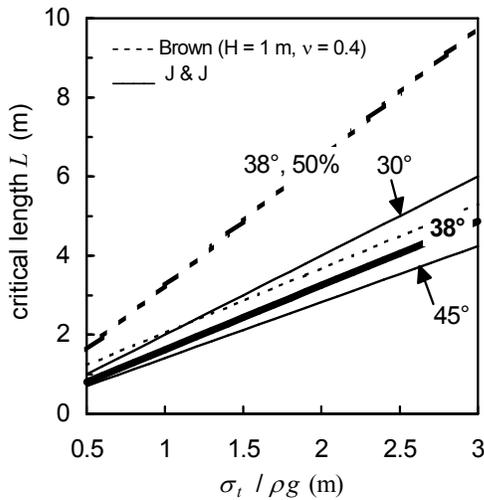


Figure 1. Critical length of fracture in weak layer (zero shear support), leading to a tension zone before crown fracture starts (Brown et al., 1972; Jamieson and Johnston 1992). Solid lines for eq. 3, dashed line shows effect of Poisson ratio and slab thickness (eq. 2). The uppermost line is an estimate for the effect of an incomplete (50%) loss of shear support (eq. 1)

Jamieson and Johnston (1992) relate the size of slab avalanches to the mechanical properties of the slab, in particular to the tensile strength. Based on a static analysis, the crown fracture occurs where the length of the tension zone attains a critical downslope length:

$$L = H \frac{\sigma_t}{\tau_{xz}} \text{ so that } L = \frac{\sigma_t}{\rho g \sin \psi} \quad (3)$$

The stronger the slab the longer is the critical length, the steeper the slope the shorter is the critical length. This findings were supported with field data.

Figure 1 shows numerical results for the critical length as defined in Eqs. 1-3, using mechanical parameters as defined in Tab. 1. Typical values are 1 to 3 m, or 3 to 8 m depending on the tensile strength of the slab (and on the amount of loss of shear support). These values can serve as a first estimate for the size of an imperfection. However, these lengths were in fact rather determined to calculate the onset of brittle tensile failure. The above equations can be modified by introducing the loss of shear stress support instead of the applied shear stress, i.e. the loss is not complete as assumed above, but some residual strength or friction exists. This leads to larger values of the critical length. This result is true for any calculation of superweak zones. The size is underestimated

since no contribution from the sides is taken into account. Furthermore the equilibrium condition is not fulfilled during the process of rapid fracture propagation, so that the lengths given above have to be considered as lower bound for the start of tensile fracture.

McClung (1979, 1981, 1987) applied a model developed by Palmer and Rice (PR) (1973) for the growth of a shear band (or slip surface) in an over-consolidated clay mass. The approach specifies that a shear band is initiated at a stress concentration in the weak layer: a slow strain softening at the tip of the band follows, until a critical length is reached, whereupon the band propagates rapidly. Two important features of the PR model are that it shows how a snow slab can fail at applied loads below the peak shear strength of the snow in the weak layer, and that it predicts delay of catastrophic shear band propagation following the time when propagation criteria are met due to bulk viscoelastic response in the slab. In contrast to other models failure can continue as the strength decreases due to strain softening. The main impediment to application of PR concepts to snow slab release is that no obvious process is available for initiation of a stress concentration in the weak layer. The PR approach is similar to a Griffith criterion, which leads to analogous results, but with the important difference that in the PR model the stress conditions at the tip of the band are non-singular. Application of these concepts points to an instability associated with the softening process leading to a critical length for the shear band. The propagation criterion is given by:

$$\frac{H(1-\nu)}{4G} \left( (\tau_g - \tau_r) \frac{L}{H} \right)^2 = (\tau_p - \tau_r) \delta \quad (4)$$

with  $\delta$  the displacement in the shear band from peak to residual stress. The term on the left is called the driving force term and provides the energy to drive the band. The term on the right provides the resistance to band extension. The model assumes that the end zone length  $\omega$  is small compared to other dimensions such as  $L$ . The critical length  $L$  for band extension can therefore be given:

$$L = \frac{H}{\tau_g - \tau_r} \sqrt{\frac{4G}{H(1-\nu)} (\tau_p - \tau_r) \delta} \quad (5)$$

Figure 2 (adapted from Conway, 1998) shows the range for the mechanical parameters defined in Table 1. A length of 5 m could be considered as typical value; however, variation is large. Figure 2 reveals some discrepancy of the model since in the limiting case when the applied shear stress is equal to the peak shear stress ( $\tau_p / \tau_g = 1$ ), the length does not become zero.

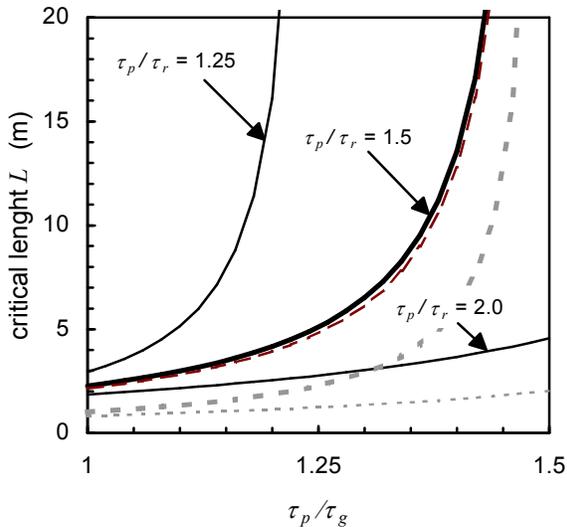


Figure 2. Critical half length  $L$  for shear band propagation (eq. 5). Typical values are used as given in Table 1. Solid lines are for  $\delta = 0.1$  mm (corresponding to a weak layer thickness of 10 mm), and  $\nu = 0.2$ ; dashed line is for  $\nu = 0.1$ . Dotted lines show lower bounds with  $\delta = 0.02$  mm (corresponding to a weak layer thickness of 2 mm) (after Conway, 1998).

McClung (1979) describes three possibilities for slab failure: (1) If no external loading (e.g. by snow fall) occurs, the softening region will progress slowly up the slope, and if it reaches a critical length, further extension would result in rapid propagation of the band. This would provide an explanation for the delayed action avalanches. (2) Provided there exists a local failure in the weak layer, under external loading conditions (snowfall), the driving force term may be rapidly increased by an increase in  $\tau_g$ . Thus the propagation criterion might be satisfied without a significant increase in the softened basal area, and an explanation of a direct action avalanche is provided via a critical stress condition. (3) Added loading by new snowfall causes applied loads  $\tau_g$  to approach the peak strength over regions of the weak layer. Under these conditions the assumptions made in deriving the propagation condition are no longer realistic, and rapid catastrophic shear fracture would be expected in the weak layer.

The size of the end region is the characteristic length  $\omega$  in which the shear stress falls from the peak to the residual value. It is a key parameter and can be considered as the minimal length to initiate any progressive failure process. Assuming a linear distribution of shear stress in the end zone, PR have given the estimate of the end zone length

$\omega$  for small end zone lengths with respect to slab length as

$$\omega = \frac{9\pi G}{16(1-\nu)} \frac{\delta}{\tau_p - \tau_r} \quad (6)$$

The end zone size can be considered as a minimum size in such a progressive failure model. If the length of the shear band is not larger than the size of the end zone  $\omega$ , shear band propagation at an applied stress less than the peak strength of the material cannot take place. In fracture mechanics the analogous situation occurs because of large-scale plasticity effects. If the crack length is of the order of the plastic zone size, propagation is not possible because of the large amount of work needed for propagation. Size effects of this type would tend to prevent small-scale flaws from propagating. Figure 3 shows that a typical value of the end zone length could be 0.7 m, again variation is large, between 0.2 and 2.2 m.

The estimates have a certain consistency with respect to rate: smaller sizes are implied for more rapid loading, consistent with results for concrete, and plausible in the light of skier triggering. However, the critical length for fracture propagation must be a multiple of the end zone size  $\omega$ . Figure 3 suggests that in the brittle range the size of the end zone is typically less than 1 m, in the order of several 10 cm.

Based on the statistical approach on snow strength established by Gubler (1978a,b), Gubler and Bader (1989) described how initial fracturing

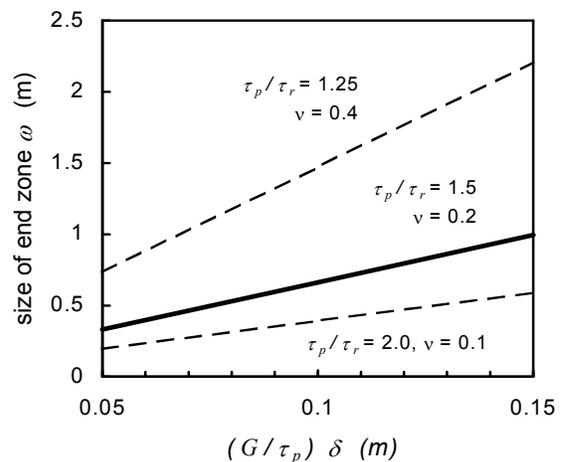


Figure 3. Estimate for size of end zone  $\omega$  (eq. 6). Typical values used as given in Table 1. For fast loading rates the value for the factor  $(G/\tau_p)\delta$  is about 0.05 m ( $G = 0.5$  MPa,  $\tau_p = 1$  kPa,  $\delta = 0.1$  mm), whereas for slow rates it is about 0.15 m ( $G = 0.3$  MPa,  $\tau_p = 2$  kPa,  $\delta = 0.1$  mm).

could happen. Local concentration of stresses, strain-rates and strains as the result of local inhomogeneities in layering, or in the vicinity of obstacles, and in low viscosity layers, leads to initial ductile failures, which eventually by coalescence can reach critical size for initiation of fracture propagation. Based on a statistical description of strength, they modelled the increase of strength (depending on time and temperature) and showed that failure is locally possible during loading periods (snowfall). This failure could serve as nucleation for slab release. However, there is no connection yet to a slab release model.

Bader and Salm (1990) introduced the term of superweak zones as deficit zones where shear stresses from overburden snow cannot or can only insufficiently be transmitted (as defined above). Such a-priori existing ground-parallel super-weak zones cause the well known stress concentrations. They conclude that without superweak zones avalanche release is highly improbable. The superweak zones are the analogue of the flaws which explain fracture in classical fracture mechanics (Griffith theory). They assume within a weak layer of thickness  $d$ , a superweak zone of length  $2L$  supporting no shear strength, representing the crack. The stress distribution is singular at the end of the superweak zone, but can be approximated by

$$\tau_{xz}^{\max} \approx \tau_{xz} \alpha \frac{L}{H} \quad (7)$$

The factor  $\alpha$  contains the properties of slab and weak layer:

$$\alpha = \sqrt{\frac{1 - 2\nu}{2(1 - \nu)}} \frac{\eta_s H}{\eta_0 d} \quad (8)$$

with the viscosity of the slab  $\eta_0$  and of the weak layer  $\eta_s$ . For  $\alpha = 1$  the common expression derived by Perla and LaChapelle (1970) results. Bader and Salm (1990) also derived the length of influence which is the distance from the end of the zone to a point where the shear stress has reached 105% of the undisturbed value. This length is needed to describe fracture propagation.

Based on the expressions for stress and strain rate at the edge of the superweak zone Bader and Salm (1990) apply a simple and rather qualitative model to study shear fracture. The fracture propagates at the crack tip with a speed  $v_r$  into the weak layer of thickness  $d$  and viscosity  $\eta_s$ . For simplicity constant shear stresses and strain rates over the length of influence are assumed. If at the crack tip the strain rate equals the critical value for ductile fracture propagation a fracture will start to propagate. For this a critical length is needed:

$$L_{cr} = \frac{H}{\alpha} \left( \frac{\dot{\epsilon}_{cr} \eta_s}{\tau_{xz}} - 1 \right) \quad (9)$$

Brittle failure will start if the strain rate at the crack tip equals the critical strain rate for brittle failure. The above equation is evaluated for some typical values. The strain rate for ductile failure propagation is assumed to be about  $10^{-4} \text{ s}^{-1}$ , and for brittle fracture propagation about  $10^{-3} \text{ s}^{-1}$ . The viscosity of the slab and the weak layer are assumed to be  $5 \times 10^8 \text{ Pa s}$  and  $0.5 \times 10^8 \text{ Pa s}$ , respectively. This values are consistent with the data summarised by Mellor (1975), and recently measured by Campo-novo (1998, unpublished research report). Campo-novo found values of the shear viscosity of  $0.2 \times 10^8 \text{ Pa s}$  to  $5 \times 10^8 \text{ Pa s}$ . His results would even justify a viscosity of  $1 \times 10^7 \text{ Pa s}$  for the weak layer. Slab depth is 0.5 m. Figure 4 (adapted from Fig. 10 of Bader and Salm, 1990) shows results for typical sets of viscosity of slab and weak layer. The effect of other parameters is minor. For a weak layer thickness of 10 mm, values for the critical length between 0.1 and 3.3 m can be found. For thicker weak layers result larger values of the critical length. This result suggests that thinner weak layers are more dangerous than thicker ones

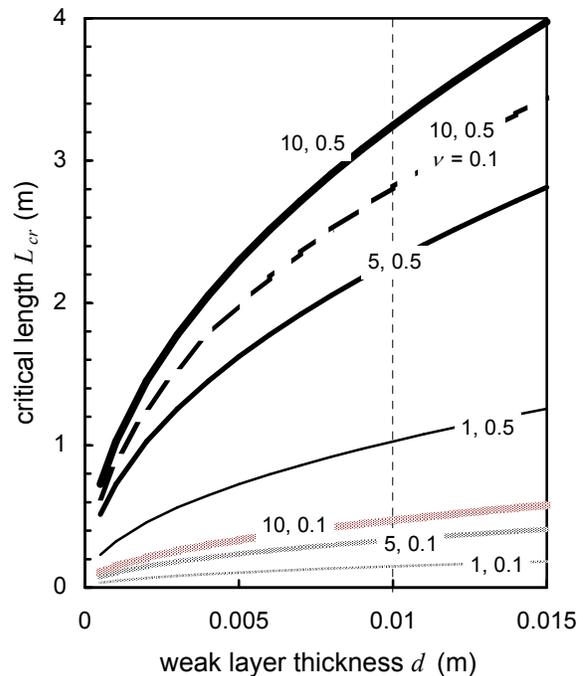


Figure 4. Critical length for ductile failure propagation (eq. 9) vs. thickness of weak layer. Numbers give values of the viscosity of the slab and weak layer respectively in  $10^8 \text{ Pa s}$ . Dashed line shows effect of Poisson's ratio (0.1 instead of 0.2 as usually used) (after Bader and Salm, 1990).

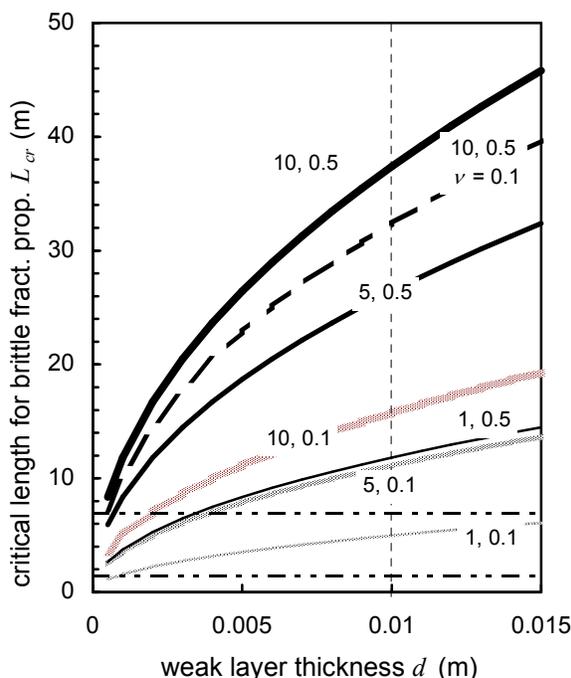


Figure 5. Critical length for brittle fracture propagation (eq. 9) vs. thickness of weak layer. Numbers give values of the viscosity of the slab and weak layer respectively in  $10^8$  Pa s. Dashed line shows effect of Poisson's ratio (0.1 instead of 0.2 as usually used). Dash-dot horizontal line shows limits for propagation based on tensile strength (2 and 10 kPa) (after Bader and Salm, 1990).

which contradicts field experience (Jamieson, 1995). Probably not the layer thickness is relevant, since the deformation is expected not to be uniform throughout the layer, but rather seems to be concentrated at the lower interface.

Bader and Salm argue that their critical value (4.4 m) is, in its order of magnitude, in accordance with estimates of Sommerfeld and Gubler (1983): 1 m, and Conway and Abrahamson (1988): 2-7 m.

Figure 5 gives an overview of the range of the critical length needed for brittle fracture propagation for typical combination of the viscosity of the slab and the weak layer. For a weak layer thickness of 10 mm values range between 5 and 37 m. Assuming typical values of the tensile strength (2-10 kPa) the extent of the snow slab, given by the tensile strength, is 1.4 to 6.9 m, independent of the slab thickness. So the values of the critical length for brittle propagation are generally larger than the limit for the crack length that can be calculated based on the tensile strength of the slab. This discrepancy to the values given by Bader and Salm (1990) is based on somewhat more realistic

values for the tensile strength from field studies (Jamieson and Johnston, 1990).

Conway (1998) explored the slip-weakening model introduced by McClung (1979) in respect to the avalanche release at the onset of rain. He relates the critical length to stability. Decrease of the slab modulus due to rain reduces the critical length, and hence promotes instability.

Nye (1975) has proposed to assess the critical nucleation size in analogy to fracture mechanics. This simple method to estimate order of magnitude size effects was discussed by Bader and Salm (1990). Assuming a bond size of 0.1 to 1 mm simple proportions yield size effects of 0.2 to 5 m.

## 5. FIELD STUDIES

In the following field studies on the strength or stability variation are reviewed. In addition field studies on acoustic emission activity that may throw some light on the slab release failure process are discussed first.

Acoustic emission measurements in the natural snowpack probably result again from microscopic fracture events that give an emission in the kHz range. Other sources frequently found are due to frictional effects within the snow and between the snow and the sensor. Coupling of the sensor is crucial. Nowadays emissions are recorded in several frequency bands, so that signals of different origin can be differentiated. For field studies the wave propagation from the source to the sensor has to be considered as well. How the wave travels through the porous snowpack seems to be controversial (Sommerfeld, 1982). This additionally complicates the interpretation of acoustic signals which in general only allow indirect conclusions about the source. From seismology it is known that the duration of the event is related to the size of failure. The larger the area where failure occurs, the longer the event. However, it seems plausible, that acoustic emission activity is an index for failure. Although, St. Lawrence and Cole (1982) report that in experiments with ice under certain conditions the acoustic emission activity could not be related to microcracking. Sommerfeld and Gubler (1983) report that they recorded signals from acoustic emission events within the natural snow cover with frequencies in the range of 10 to 100 Hz, and interpret that as the result of a failure process (relaxation area) with a typical size of the order of 0.1 to 1 m.

Field studies on the spatial variability of snow strength have been done primarily in New Zealand (Conway and Abrahamson, 1984, 1988), Switzerland (Föhn, 1989) and Canada (Jamieson, 1995). It has to be pointed out that in all studies the methods applied are far from being perfect, but in most cases appropriate and the best available.

Conway and Abrahamson (1984, 1988) have done contiguous point measurements (spacing between measurements of 0.6 to 0.9 m) of the shear strength along the crown of 5 slab avalanches. They found substantial strength variations. When the sample failed during preparation this measurement was assigned arbitrarily a stability value smaller than one. Analysing the variations a typical length can be derived. Conway and Abrahamson found the typical variation (called wave length) to be about 0.5 to 3 m. Some of the rather large variation in strength is probably due to inherent experimental variation (usually strength measurements are repeated and averaged) and the minimal wavelength found also reflects the measurement interval. Despite some of the drawbacks of these measurements, in particular the fact that failed experiments were recorded as results with stability smaller than one, the conclusions in general holds that if the pattern observed would be typical the primary concern for stability assessment would be to evaluate the variance and statistical correlations, as well as the mean value. A stability evaluation would be done by several tests that cover a certain distance (likely one or more square meters).

Föhn (1989) used the rutschblock test that he had introduced as stability test (Föhn, 1987) to study the spatial variability of snow stability and did also shear frame measurements. He found no evidence for small deficit zones that could be responsible for stability. Snow strength (stability) varied with the same order of magnitude (15-30%) as other snow parameters. Of course field studies on potential avalanche slopes could only be done during periods of intermediate to high stability, or along the fracture line of released avalanches.

The most complete field studies have been done by Jamieson and co-workers (Jamieson, 1995). On many avalanche slopes at rather low, intermediate and high stability series of rutschblock tests have been made. Jamieson (1995) reports nine cases of series of rutschblock tests (with 20 to 81 rutschblocks per series) covering avalanche slopes. The median score of a series was once 3, five times 4, twice 5, and once 6, indicating a pretty wide range of average snow stability found. The results show that most rutschblock scores can be expected to be within  $\pm 1$  step of the slope median.

No deficit zones have been found. However, scores two steps above the slope median can infrequently occur. This misleading information is often due to the fact that the operator's skis have penetrated almost to, or through, the weak layer during a rutschblock test. In addition, Jamieson (1995) clearly points out that sites near the top of slopes, near trees, over rocks and at pillows of wind deposited snow sometimes exhibit rutschblock scores quite different from the rest of the slope. Jamieson (1995) found a similar relation between rutschblock score and avalanche activity as Föhn (1987). He also compared the skier stability index, estimated based on shear frame measurements from a single pit, with avalanche activity. He found that the skier stability index  $S_K$  is a good predictor for skier triggering: values of  $S_K < 1$  indicate instability (likely triggering), values of  $S_K > 1.5$  indicate stability (unlikely triggering). However, the stability index  $S_N$  for natural triggering is less conclusive. Critical values are 2-3 for  $S_N$ . Jamieson (1995) concludes that this indicates that the critical stress failure criterion upon which  $S_K$  and  $S_N$  are based, is effective for skier-triggered avalanches, but not for natural avalanches.

Birkeland et al. (1995) measured snow resistance on four potential avalanche slopes using a digital resistograph. They found that the average resistance showed a variation of 28-58%, whereas the variation of the snow depth was 13-30%. For two cases they found a relation between snow depth and average resistance. They did not relate resistance to snow stability. However, in general, average resistance cannot be directly related to snowpack stability. In addition, this method giving average resistance is rather appropriate to find weak spots, and not superweak zones.

Summing up, there has been clearly shown that snow strength and hence stability, as other parameters (density, snow depth) vary on avalanche slopes. But there is no evidence for small deficit zones from field studies by Föhn (1989) and Jamieson (1995). This finding is even supported by Conway and Abrahamson (1984) considering their arbitrary interpretation of test results.

## 6. ARTIFICIAL RELEASE: SKIER TRIGGERING

Although the release mechanism should principally coincide, it is convenient to consider natural and artificial release separately. The reason for that is that skiers or other human causes directly induce a stress concentration within a short time over a certain area. For the skier, field measurements (Schweizer et al. 1995a,b; Camponovo and

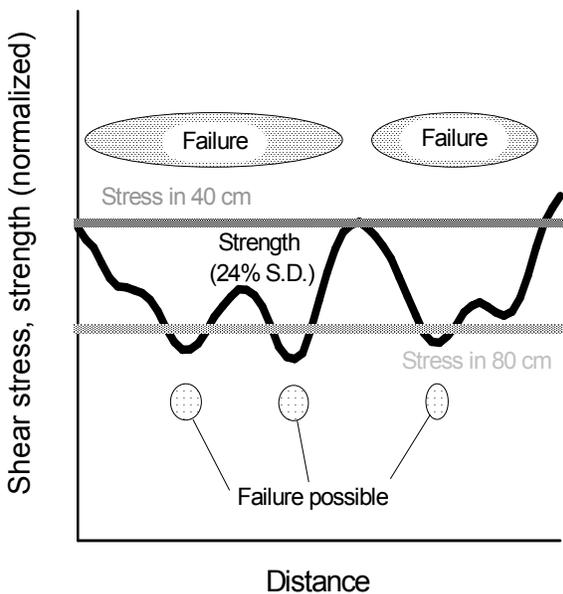


Figure 6. Conceptual sketch: Decreasing (spatial) release probability with increasing depth of weak layer. The two horizontal lines indicate the shear stress, including the stress approximately induced by a skier, for two different depths of the weak layer. It is assumed that at both depth the same weak layer of same shear strength (relatively) exists (stress and strength are normalized to the shear stress due to the slab). If the weak layer is e.g. at about 40 cm depth, the figure shows that the stress nearly everywhere is larger than the strength, i.e. that a skier might induce a local failure nearly everywhere on the slope, and hence an avalanche release is likely. For a weak layer depth of e.g. about 80 cm, the additional skier stress is relatively small (about half the value compared to the upper layer), and therefore it is rather unlikely, that a skier will trigger a slab avalanche. In the simplified example shown there are only two or three locations where a skier might induce a local failure, and, if the failure spreads out at all, an avalanche might be released.

Schweizer, 1997) indicate that the dynamic impact of a skier is of the same order as the shear stress due to the slab, covers typically less than 1 m<sup>2</sup>, and occurs within a short time, typically within 0.03 to 0.3 s. The transmission of the additional stress due to the skier within the snow cover strongly depends on the layering, and decreases with increasing depth. For a new snow slab with a depth of about 50 cm the additional dynamic stress is about 0.5 kPa. For deep slabs (> 1 m) the additional stress becomes very small.

Based on these measurements, it can be concluded that for intermediate slabs, the skier slab thickness is most frequently 40 to 60 cm (Jamieson and Geldsetzer, 1996), the skier might

induce a brittle fracture that might propagate and release an avalanche. As in the case of natural release the initial failure area has to have a minimal size for fracture propagation, and propagation can principally be fast or slow. Since the area where failure is induced by a skier is of the same order of magnitude as the estimate for the size of imperfection, the condition for fracture propagation might frequently, but not always, be fulfilled. For deep slabs triggering becomes incidental, strongly depending on the spatial variation of the snow stability. Therefore it looks like triggering would be random. However, for the majority of cases with intermediate slab thickness and low to intermediate stability, inducing a failure is frequently possible on the whole slope. This idea is illustrated with Fig. 6.

## 7. DISCUSSION AND CONCLUSIONS

The reviewed snow slab failure models give plausible explanations for naturally released, delayed action avalanches. All models were probably developed with the view to explain how avalanches can be initiated in the case where the shear stress does not reach the shear strength. They all use a continuum approach and are based on an (energy) balance approach considering a crack or shear band within a homogeneous layer. A reasonable, but very limiting simplification. The stress concentrations at the edges cause failure and drive fracture propagation, from ductile to brittle. A critical size for fracture propagation can be given. Virtually any value for that length can be calculated with reasonable assumptions. However, typical values can be given, but still roughly cover two orders of magnitude of 0.1 to 10 m. The lower range (0.1 m to 1 m) is rather associated with slow propagation, the higher with fast propagation (1 to 10 m). The model developed by McClung (1979) includes most realistic assumptions considering the mechanical properties of snow. Bader and Salm (1990) primarily explore fracture propagation, without giving reference to previous work, e.g. by McClung. All models include the slab *and* the weak layer which is the right view of the problem.

From field measurements no direct evidence for small deficit zones could be found up to now. Either the methods or situations were inappropriate, deficit zones have not been present, or do not exist at all.

Despite some plausible explanations, the models are hard (or impossible?) to verify. However, considering the simplifications involved, the wide range of reasonable values, and the fact that

there is no direct evidence from field measurements, it is questionable whether any numerical value on the size of superweak zones should be given, and arguments build on. The best estimate at present seems to be in the order of a fraction of a meter. Nucleation seems to be closer to the micro-scale than to the macro-scale.

Virtually all models assume that the imperfection to start with a priori exist. Hardly any ideas are presented how this deficit zones come into being. It seems clear, in particular in comparison with lab experiments, that with a stabilizing process such as fracture self-arrest and sintering, a shear deficit would heal again, most likely within minutes or hours. Superweak or deficit zones are probably a highly transient phenomenon. They might form frequently shortly after storms and with decreasing frequency when stability increases. Considering the time scale of sintering and ductile failure the lifetime of a deficit zone is supposed to be in the order of hours, at the most.

Once the continuum approach is left behind, and strength is seen statistically, a priori existing superweak zones of a few meters in size become unnecessary. Failure starts from damage. A statistical approach would not only explain failure initiation in weaker zones but also fracture arrest or healing (sintering) in stronger zones.

For direct action avalanches which represent the vast majority of avalanches, a simple stress criterion is likely to be sufficient, considering that the typical initial strength of persistent weak layers is less than 500 Pa.

As skiers impart substantial stress concentrations within a short time, likely to cause brittle failure, a simple stress criterion seems to be sufficient as a first approximation, as well. This is strongly supported by the fact that the skier stability index seems to be a good predictor of skier triggered avalanches. However, slab properties and propensity for fracture propagation are crucial as well, but not considered in the simple stress criterion. Based on field measurements it is concluded that an imperfection is not necessary for skier triggering, in particular not for intermediate slab thickness (0.5 m) and low to intermediate stability. For deep slabs or in general for higher stability, skier triggering, considering strength or stability variations, starts to look like a random process, since triggering is limited to some spots with lower strength (Fig. 6). Of course, under any conditions, if stress concentrations by a skier accidentally coincide (in space and time!) with a zone of lower strength instability is more easily achieved. However, considering that a superweak zone is a highly transient phenomena, the coincidence must be very, very

rare, much rarer than observed skier triggered avalanches.

Although plausible explanations exist, there is still a lack of comprehensive physical understanding of the failure initiation and release process. Future research might focus on the nucleation (failure initiation) problem which seems to be most unknown. Laboratory experiments on the failure process might throw some light on that and at the same time provide data to feed snow and slab failure models. A link between the micro-scale (snow failure) and the macro-scale (slab failure) would be needed. A statistical approach considering disorder at different scales seems to be most promising. But even then it is questionable whether laboratory experiments will be sufficient. To study conditions for fracture propagation, experiments on the macro-scale, in the field are probably necessary. "One to one" verification of snow slab models in the field seems to be the final challenge.

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