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The temperature dependence of the effective elastic shear modulus of snow

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Abstract

In order to determine the temperature dependence of the effective elastic properties of snow, dynamic torsional shear experiments at a frequency of 1 Hz were performed in a cold laboratory with a stress-controlled rheometer. Two types of natural snow samples of small rounded or faceted grains with partly decomposed and fragmented particles (mean size 0.4–0.5 mm) with densities in the range of 220–250 kg m^{-3} were tested. Results indicate that snow is a rheologically simple material, so that the data can be normalized for analysis. The mean effective elastic shear modulus G' at -10 °C was found to be about 0.75 MPa for the density range and snow type tested. The modulus decreases with increasing temperature following an Arrhenius relation below -6 °C with an apparent activation energy of about 0.2 eV. Between -6 °C and the melting point, the decrease is much stronger showing that the modulus is highly temperature-sensitive in the range of interest for snow slab stability evaluation. An empirical relation is fitted to the data over the whole range of temperatures to provide a single smooth relation for engineering applications.

Keywords: Snow mechanical properties; Snow deformation; Temperature effect

1. Introduction

Assessing the effect of temperature and its variation on snow stability has been described by de Quervain (1966) as one of the most challenging problems in avalanche forecasting. A few decades later, this statement is still valid. A prerequisite for the understanding of the complex effects of temperature on snow stability is the knowledge about the temperature dependence of the relevant mechanical properties.

McClung and Schweizer (1999) proposed that shear failure toughness and stiffness (initial tangent modulus) of snow would be the most important mechanical parameters to consider for the dry-snow slab release problem. Both seem to be highly temperature-sensitive. Schweizer (1998) has in fact shown that stiffness is the most temperature-dependent parameter with strength only slightly temperature-dependent. The elastic properties are particularly important for rapid loading processes such as skier triggering both for the amount of deformation exerted and the fracture propagation (Schweizer, 1999). Strength in the brittle range seems to be independent of temperature in accordance with measurements of the toughness of ice (Goodman, 1980), likely due to the fact that the sintering process is not efficient.

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Determining the elastic modulus involves difficulties. Mellor (1975) has already pointed out that the initial tangent modulus from quasistatic tests may be inherently deceptive, except if the tests are run at high strain rates. Otherwise, the initial tangent modulus often includes non-elastic parts of deformation. Accordingly, Mellor (1975) estimates that also the true effect of temperature on the modulus would be hard to determine and proposes that the best estimates could likely be obtained from high frequency vibration experiments.

Sinha (1978a,b) describes the same problem for determining the elastic modulus of ice, the matrix material of snow. He reports that values of the modulus obtained by static methods, e.g. by determining the initial tangent modulus from creep experiments, usually are too low compared to ones obtained by dynamic methods. Static values would usually represent an effective modulus that includes not only elastic, but also viscous parts of deformation. Accordingly, reported values of the Young's modulus are frequently not based on purely elastic deformation. This seems to be the case for the values of Young's modulus of snow compiled by Mellor (1975) that vary between about 1 and 200 MPa for a snow density of 200 kg m^{-3}. Part of the variation in the results of e.g. Schweizer (1998) is, however, due to the inherent variation between natural samples from a study plot. The statically determined values from direct simple shear tests by Schweizer (1998) increase with increasing strain rate in line with the discussion above, indicating that the values obtained at lower rates include viscous parts of deformation.

The temperature dependence of the Young's modulus of snow is suggested to be small in the range of interest, a few percents only (Mellor, 1975). This is in accordance with the behaviour of ice (Petrenko and Whitworth, 1999).

The temperature dependence of ice deformation can be described with the Arrhenius equation up to about -10 °C (Hooke et al., 1980):

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \exp\left(-\frac{Q}{RT}\right) \tag{1}$$

where $\dot{\varepsilon}$ is the strain rate at absolute temperature T, $\dot{\varepsilon}_0$ is a reference strain rate, Q is the activation energy for creep and R is the gas constant. This means that the deformation process of ice is thermally activated with an activation energy of $Q \approx 0.62$ eV corresponding to the activation energy for self-diffusion. Above about -8 °C, the Arrhenius relation breaks down. The increase of deformation with increasing temperature becomes much stronger, ice starts to deform more easily as it approaches the melting temperature. There have been suggestions that this is due to the presence of water in ice, and it may be related to pre-melting phenomena at the grain boundaries. However, the interpretation of the deformation processes acting as the melting point is approached, is not yet well developed (Petrenko and Whitworth, 1999). The use of an Arrhenius relation for laboratory work at temperature below -10 °C is reasonable, as this relation is based on sound physical principle. However, use of Eq. (1) as an empirical representation of temperature dependence at temperature above about -10 °C should, according to Hooke et al. (1980), be discontinued, and more accurate empirical relations representing the curvature in plots of ln ε vs. $1/T$ should be sought. Accordingly, Smith and Morland (1981) fitted the data of Mellor and Testa (1969) to several empirical functions.

The same behaviour as described above for ice has also been suggested for snow. Based on the measurements of Narita (1983), Kirchner et al. (2001) have shown that the apparent activation energy increases in the high-temperature range. However, only data for three different temperatures were analysed.

Sinha (1978a,b) undertook a study of the elastic/delayed elastic behaviour of ice under conditions of constant stress at small strains. He proposed that the deformation of ice would consist of at least three strain contributions: elastic, delayed-elastic and viscous, and could be described by the following empirical relation:

$$\varepsilon_t^\sigma = \frac{\sigma}{E_0} + c\left(\frac{\sigma}{E_0}\right)^s \left\{1 - \exp\left(-[a_T t]^b\right)\right\} + \dot{\varepsilon}_1 \left(\frac{\sigma}{\sigma_1}\right)^n_t \tag{2}$$

where ε_t^σ = strain for stress, σ, and time, t; E_0 = Young's modulus (time-dependent and, for ice, only slightly temperature-dependent); a_T = temperature-dependent factor = $a_0\exp(-Q/RT)$; Q = activation energy; R = gas constant; T = absolute temperature (in Kelvin); $\dot{\varepsilon}_1$ = viscous creep rate for unit stress; and a_0, c, s, b, n, are factors initially assumed to be constant. The first term on the right hand side is the initial elastic deflection, the second term describes the delayed-elastic (recoverable) strain; the third term is the viscous flow. Sinha found the above equation a good description of experimental results for $n=3$, $b=0.34 \approx 1/n$ and $s=1$. The elastic and delayed elastic contributions are fully recoverable. Whereas the elastic contribution is not temperature-dependent, the delayed-elastic and viscous part are highly temperature-sensitive. Accordingly, Sinha (1978a) concludes that it would be relevant to closely control the experimental conditions and avoid determining the effective modulus and its dependence on temperature with experiments where the viscous part of deformation dominates.

Furthermore, Sinha's (1978b) analysis indicates that ice is a thermo-rheologically simple material, i.e. that the effect due to time (strain rate) and temperature can be combined into a single parameter through the concept of the "time–temperature superposition principle" (Findley et al., 1976). Sinha (1978b) describes the temperature shift factor as:

$$-\ln a_T = \frac{Q}{R}\left(\frac{1}{T}\right) + d \tag{3}$$

where d is a constant. If a_{T-1} and a_{T-2} are the values corresponding to T_1 and T_2, respectively, Eq. (3) gives:

$$\ln\left(\frac{a_{T-2}}{a_{T-1}}\right) = \frac{Q}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \tag{4}$$

Thus, in Sinha's notation the constant a_T incorporates the shift function.

Accordingly, and since the temperature dependence of the dynamic elastic shear modulus G' can be assumed to follow an Arrhenius relation for low temperatures:

$$G' = G'_0 \exp\left(-\frac{Q}{RT}\right), \tag{5}$$

experimental values of the shear modulus can be normalized to e.g. $T_R = 263$ K by using the following relation:

$$\frac{G'\ (T=T_1)}{G'_R\ (T=T_R)} = \exp\left(\frac{Q}{R}\left(\frac{1}{T_R} - \frac{1}{T_1}\right)\right). \tag{6}$$

The aim of the present study is to determine the true temperature dependence of the effective elastic shear modulus, i.e. of the recoverable elastic/delayed-elastic part of snow deformation. This effective elastic modulus is highly relevant for the dry snow slab release problem. In addition, it has been proposed that the elastic modulus has substantial potential as a snow mechanical index property (Shapiro et al., 1997) which in part has been shown by Camponovo and Schweizer (2001). In order to comply with the experimental conditions as described above, the newly developed rheometric approach is used, performing low strain dynamic torsional shear experiments at high loading rate (1 Hz) (Camponovo and Schweizer, 2001).

2. Methods

Torsional, dynamic shear tests were performed in the cold laboratory at Weissfluhjoch (Davos, Switzerland). The snow samples (60 mm in diameter, about 7 mm in height) were cut from natural snow blocks from the Weissfluhjoch study plot and held between two grooved plates. Whereas the bottom plate is fixed, the upper plate rotates and applies the shear force. Dynamic (oscillation) and static (e.g. creep) experiments can be performed. The rheometer, a Bohlin CVR 50, has high precision and resolution of the

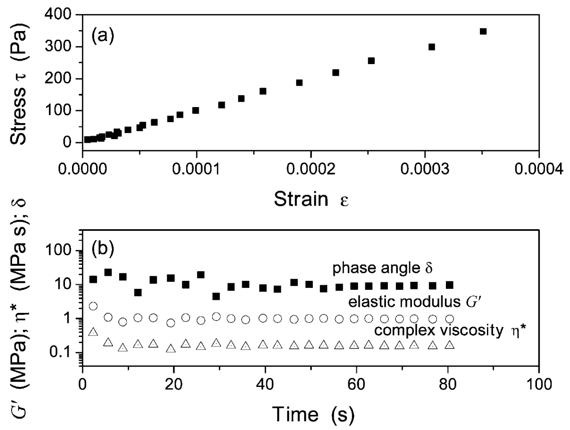

Fig. 1. Results of an amplitude sweep experiment (#m_n328, Table 3). (a) Stress–strain plot (top). (b) Elastic or storage shear modulus G', complex viscosity η^*, and phase angle δ shown vs. time (bottom). The applied stress increases logarithmically. The test is stopped before the elastic modulus G' starts to decrease, in the case shown at $\tau=348$ Pa and $\varepsilon=3.5\times10^{-4}$. The low value of the phase angle $\delta \approx 9°$ indicates that the deformation is mainly elastic.

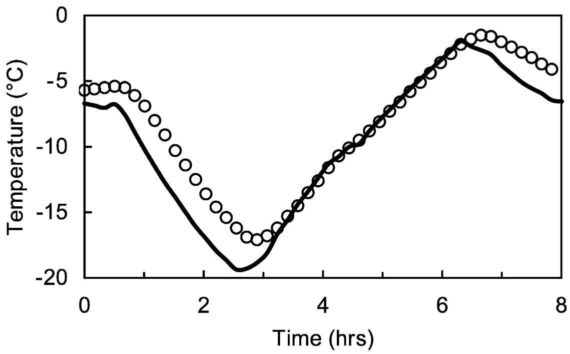

Fig. 2. Temperature cycle during continuous oscillation testing (#m_n323, Table 3). Snow sample temperature (solid line) and apparatus temperature (dots) shown vs. time. Sampling is every 10 min when a continuous oscillation test is performed.

torque at relatively low stress. Consequently, the material response, at a small applied force, can be measured with high temporal and local resolution. The results of oscillation experiments can be used to easily split the deformation into the elastic and the viscous part. This has been shown by Camponovo and Schweizer (2001), who have introduced the rheometric method for snow and described it in more detail.

The basic idea for determining the temperature effect is to imply a small strain (or alternatively stress), in oscillatory manner, so that the texture of the snow sample is not changed and the test is fully reproducible, and then to successively change the temperature of the sample (by changing the temperature of the cold lab), and to repeat the test until many measurements for the temperature range of interest are performed.

2.1. Amplitude sweep

Before each series of temperature measurements, the sample is first characterized with an amplitude sweep test at a frequency of 1 Hz and the strain at the proportional elastic limit is determined (Camponovo and Schweizer, 2001). During the amplitude sweep test, the frequency of oscillation is kept constant (1 Hz), whereas the stress applied increases logarithmically from 10 to 800 Pa in 30 steps (Fig. 1). The elastic limit strain is the strain below which the strain is proportional to the stress, i.e. no changes in structure occur and the test is fully reproducible as shown by Camponovo and Schweizer (2001). In an amplitude sweep, beyond the proportional elastic limit, the storage modulus G' starts to decrease, i.e. the strain is no longer proportional to the stress. Accordingly, for our purpose the test is stopped just before the modulus tends to decrease. Based on the results of the amplitude sweep test the strain or stress limit to be used during the temperature measurements is determined.

Previously we tried to prescribe the stress value to be reached during the continuous oscillation test, but it turned out to be more reliable to prescribe the strain ε to be reached. By doing so the strain does not become large at high temperatures, near the melting point, where the material is much weaker. Thus, the proportional elastic limit strain is not exceeded and the snow sample is not damaged.

2.2. Continuous oscillation during temperature cycle

Once the sample is characterized with an amplitude sweep test and the proportional elastic limit strain is known, the sample is successively tested with a continuous oscillation (single frequency) test at varying temperature. Again the frequency is held constant (1 Hz) and the torque applied is such that a prescribed strain is reached. The continuous oscillation test takes a few seconds and is repeated every 600 s while the temperature changes. Usually 48 tests have been made during a complete temperature cycle lasting 8 h. During this time the lab temperature was first decreased to -20 °C from its initial value of about -7 °C, and then increased to about -1 °C, and finally decreased

Table 1
Characteristics of snow layers from which samples were prepared

Sampling date	Grain type	Grain size (mm)	Hand hardness index	In-situ temperature (°C)	In-situ density (kg m^{-3})	Ram hardness (N)
3 December 98	4, (3), (2)	0.25–1	1–2	-10.5	225	< 10
12 January 99	3, (2)	0.25–0.5	2	-6.3	215	< 15

Grain type is given according to the International Snow Classification (Colbeck et al., 1990); (2) decomposing and fragmented precipitation particles, (3) small rounded grains, (4) faceted crystals. Grain types in brackets indicate secondary or tertiary frequency.

towards the initial value (Fig. 2). The temperature was measured by the rheometer internally and externally, close to the sample. Preliminary tests have shown that the latter one is closer to the actual snow sample temperature and is therefore used for the analysis. It is typically about 1 °C lower and not lagged compared to the internal temperature measurement (Fig. 2).

2.3. Snow sampling

All measurements were performed with snow samples cut out of snow blocks taken from the study plot at Weissfluhjoch at two occasions: 3 December 1998 and 12 January 1999. Samples were cut parallel to natural snow layering. Table 1 compiles in-situ characteristics of the two snow types used at the time of sampling. Both layers were recently deposited snow. The first layer was 6 cm thick and 13 cm below the surface, the second one 8 cm thick and 23 cm below the snow surface. The layer from 3 December 1998 was subject to a large temperature gradient of 27 °C m^{-1} at the time of sampling. After sampling, the snow blocks were stored up to 19 days at −36 °C to avoid further metamorphism.

Samples were analysed in the cold laboratory with different methods in order to accompany the mechanical properties determined below with relevant structural properties. Pictures of disaggregated samples have been taken and analysed as described by Baunach et al. (2001). Surface sections of samples preserved in

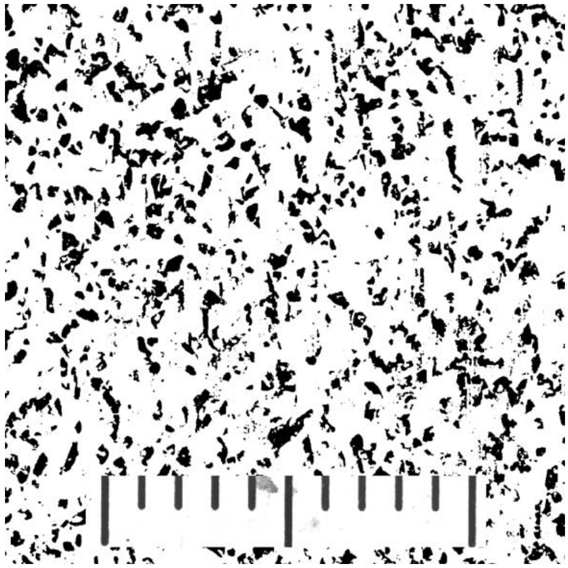

Fig. 4. Surface section of the snow sample taken on 3 December 1998. Scale shown is 1 cm.

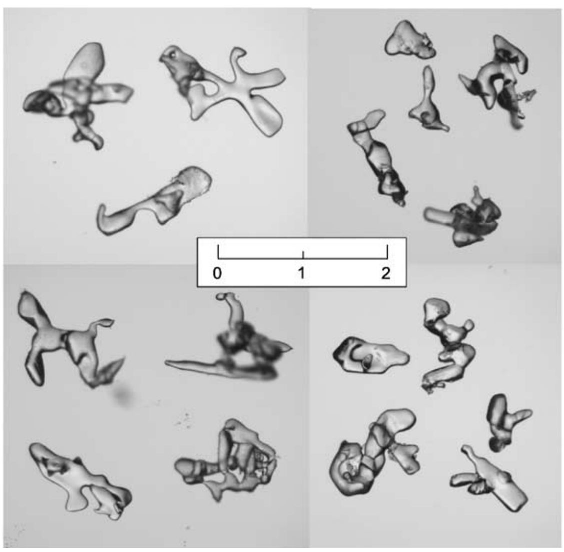

Fig. 3. Macrophotographs of disaggregated grains from snow sample taken on 12 January 1999. Scale shown is 2 mm.

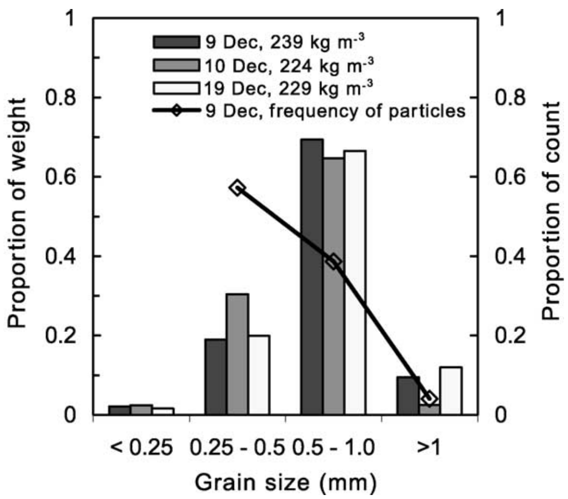

Fig. 5. The grain size of three samples cut out of the snow block collected at 3 December 1998 has been analysed by sieving. Bars indicate frequency by weight. Line shows frequency by count for the sample tested at 9 December 1998 only. The particles in the 0.25-mm sieve were not counted. Average grain size is 0.71 mm by weight and estimated to be 0.2–0.4 mm by count.

Table 2
Mean diameter of snow samples taken at two occasions (3 December 1998, 12 January 1999) determined with different methods

Method	Mean diameter (mm) (3 December 1998)	Mean diameter (mm) (12 January 1999)
Field observation	0.25–1	0.25–0.5
Sieving (by weight)	0.71	–
Sieving (by count)	0.2–0.4	–
Image analysis	0.5	0.39
Surface section	0.28	–

diethyl phtalate were prepared as described by Good (1987). Sieving of samples through a stack of three sieves with mesh openings of 1, 0.5 and 0.25 mm followed the procedures given by Sturm and Benson (1997). A relation of grain mass vs. sieve mesh size derived by Bader et al. (1939) and refined for the case of depth hoar crystals by Sturm and Benson (1997) was applied to estimate the number of particles that passed the sieve with the finest mesh (0.25 mm).

3. Results and discussion

Sample characterization in the cold laboratory by taking pictures of disaggregated grains (Fig. 3), by surface section analysis (Fig. 4) and by sieving (Fig. 5) essentially confirmed the initial characterisation in the field. Table 2 compiles the mean diameter for the two samples found with different methods. Image analysis of digitized macrophotographs of disaggregated samples for the two dates showed that the mean diameter based on the area equivalent is 0.5 mm for the sample taken at 3 December 1998, and 0.39 mm for the sample taken on 12 January 1999 (Table 2). The frequency distributions are given in Fig. 6.

For the sample of 3 December 1998, the sieving analysis showed that in fact 90% of the grains were in the range of 0.25–1 mm, if considering weight. The average grain size by weight was 0.71 mm. In addition, the particles were counted. The mean diameter by counting was about 0.2–0.4 mm. The uncertainty is due to the fact that the smallest particles that passed the 0.25-mm sieve could not be counted.

Within 3 weeks after snow sampling, tests were performed. Each test consists of an amplitude sweep and a continuous oscillation during a temperature cycle. Table 3 lists the measured parameters for 13 tests that were analysed. Due to experimental problems some other results were discarded. Some of the 13 tests can only be analysed for certain temperature ranges, since the strain sometimes exceeded the elastic limit close to the melting temperature, contact was lost towards the melting point or the sample melted. 10 tests were performed with samples from the snow taken at 3 December 1998 (type 1), the other three from snow taken at 12 January 1999 (type 2).

Fig. 7 shows the results of two tests clearly demonstrating that the elastic modulus decreases with increasing temperature with a much accelerated decrease towards 0 °C. For one of the experiments in Fig. 7, it is shown that there is virtually no hysteresis during a temperature cycle (Fig. 2) again proving that the applied strain is recoverable.

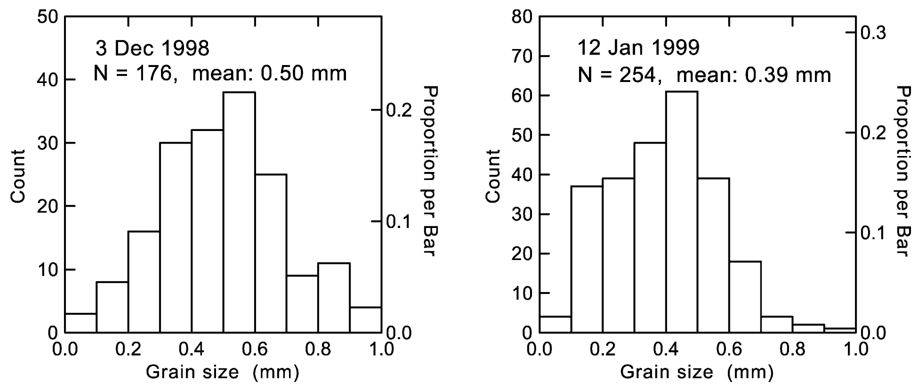

Fig. 6. Frequency distribution of grain size (diameter) of the two snow samples taken on 3 December 1998 (left) and 12 January 1999 (right) determined from macrophotographs of disaggregated samples with image analysis procedures. Diameter given is calculated from area equivalent.

Table 3
Summary of results

Test no.	Snow type	Density (kg m^{-3})	Amplitude sweep no.	Temperature (°C)	Shear modulus G' (Pa)	Complex viscosity η^* (Pa s)	Elastic limit strain ε_t	Temperature experiment no.	Initial temperature (°C)	Initial shear modulus G' (Pa)	Slope c (K)	Intercept (Pa)
1	1	233	m_n318	−9.7	8.66×10^{5}	1.39×10^{5}	0.9×10^{-4}	m_n319	−9.6	8.64×10^{5}	1232	8232
2	1	239	m_n320	−7.6	8.01×10^{5}	1.30×10^{5}	0.9×10^{-4}	m_n321	−7.1	8.01×10^{5}	2203	219
3	1	238	m_n322	−7.2	9.89×10^{5}	1.59×10^{5}	0.9×10^{-4}	m_n323	−6.7	9.90×10^{5}	1801	1198
4	1	249	m_n324	−7.5	9.10×10^{5}	1.46×10^{5}	0.9×10^{-4}	m_n325	−7.2	9.20×10^{5}	1677	1757
5	1	238	m_n326	−7.4	7.44×10^{5}	1.20×10^{5}	0.9×10^{-4}	m_n327	−7.7	7.50×10^{5}	2293	139
6	1	246	m_n328	−7.6	9.87×10^{5}	1.59×10^{5}	0.9×10^{-4}	m_n329	−7.6	9.77×10^{5}	1858	982
7	1	237	m_n334	−10.6	1.01×10^{6}	1.62×10^{5}	0.9×10^{-4}	m_n335	−10.2	9.68×10^{5}	2205	217
8	1	244	m_n338	−6.6	6.69×10^{5}	1.08×10^{5}	0.9×10^{-4}	m_n339	−6.4	6.44×10^{5}	3029	7
9	1	242	m_n343	−8.6	6.75×10^{5}	2.19×10^{6}	0.9×10^{-4}	m_n344	−8.5	6.96×10^{5}	2219	174
10	1	247	m_n345	−8.8	5.59×10^{5}	1.81×10^{6}	0.9×10^{-4}	m_n346	−8.9	5.75×10^{5}	2784	17
11	2	219	m_n361	−8.3	7.84×10^{5}	1.28×10^{5}	0.9×10^{-4}	m_n362	−8.2	7.49×10^{5}	2716	28
12	2	223	m_n363	−5.3	6.62×10^{5}	1.07×10^{5}	0.9×10^{-4}	m_n364	−5.5	6.77×10^{5}	2925	11
13	2	219	m_n365	−8.9	7.20×10^{5}	1.18×10^{5}	0.9×10^{-4}	m_n366	−8.8	7.57×10^{5}	2484	62

Snow type of 3 December 1998 is designated as type 1, of 12 January 1999 as type 2. Slope and intercept of linear fit below −6 °C are given (Fig. 9).

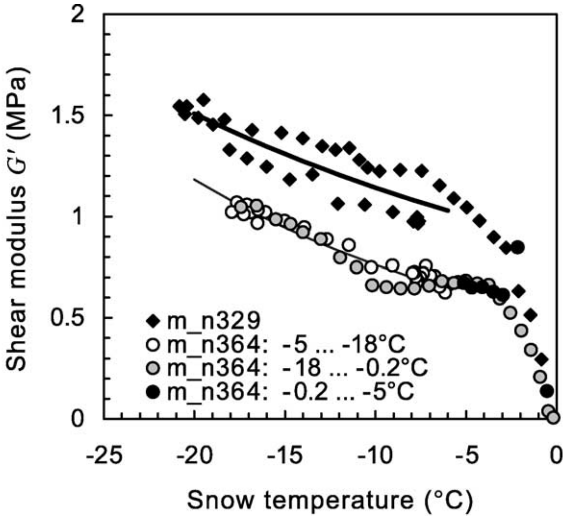

Fig. 7. Temperature dependence of the effective elastic shear modulus G'. Results of two tests shown. For one of the tests the data points are shown for three different parts of the temperature cycle. Lines indicate fit assuming an Arrhenius relation (see Fig. 8).

Assuming that below -10 °C the increase is following an Arrhenius relation (Eq. (5)), the modulus should be proportional to the inverse absolute temperature T when plotted on a semi-logarithmic scale. As can be seen in Fig. 8, the decrease follows in fact an Arrhenius relation up to about -6 °C. In Table 3, the slope of the linear relationship, c, for each of the tests is given, and shown in Fig. 9. The mean value of the slope below -6 °C is $c = -2.26 \times 10^3$ K.

The fact that all slopes are very similar shows that snow is, at least below about -6 °C, a rheologically simple material. Deriving the shift function from the average slope, the curves of all results can be normalized to e.g. $T_R = 263$ K by using Eq. (6). This is demonstrated in Fig. 10 for the same experiments as shown before and an additional experiment, yet now normalized to $T_R = 263$ K. The agreement between the curves is clearly improved.

Normalizing all values of the modulus G' as given in Table 3 to the reference temperature of $T_R = 263$ K shows that the reproducibility between the amplitude sweep test and the subsequent continuous oscillation test is very good (Pearson correlation coefficient: 0.99). The mean effective shear modulus G' at the reference temperature $T_R = 263$ K is 0.75 MPa. For the narrow range of density, no significant dependence of the modulus on density shows up.

Assuming that the temperature relation below about -6 °C follows an Arrhenius relation, the slope c is related to the activation energy (Eq. (5)). The mean slope determined above corresponds to an apparent activation energy of $Q = 0.20$ eV. Above -6 °C, the apparent activation energy increases strongly.

In order to describe the temperature dependence above -6 °C, the data normalized to the reference

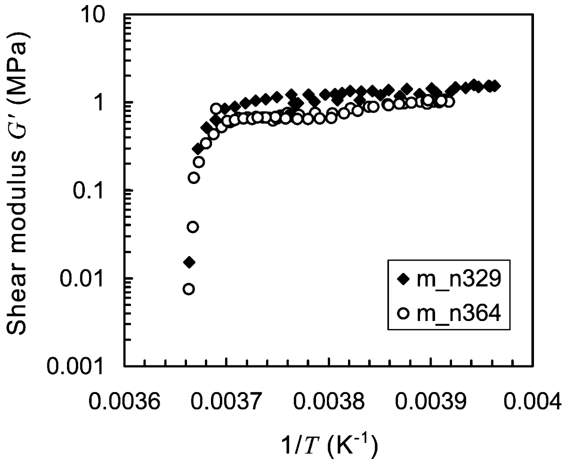

Fig. 8. Temperature dependence of the effective elastic shear modulus G'. Results of two tests shown, same as in Fig. 7, but shown as semi-logarithmic plot with the inverse absolute temperature as independent variable.

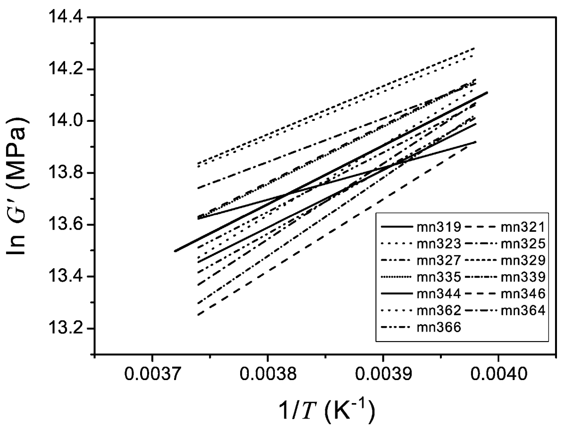

Fig. 9. Linear regression lines for data below -6 °C for all 13 experiments. Bold line indicates mean line: $\ln G' = 2263/T + 5.08$. The slope of the mean line corresponds to an apparent activation energy of 0.20 eV.

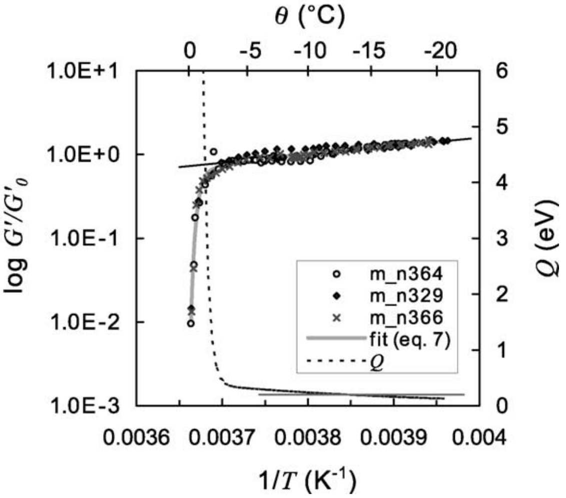

Fig. 10. Normalized effective elastic shear modulus G' vs. inverse absolute temperature. Thin line indicates average slope of all experimental results below -6 °C. Bold line gives fit to data according to Eq. (7) over the whole temperature range for the three tests shown. Derivative of fit (dashed line) is apparent activation energy (in eV).

temperature $T_R = 263$ K is fitted to a single smooth curve for the whole temperature range:

$$\ln\frac{G'}{G'_0} = A_0 + A_1\exp\left(\alpha_1\left(\frac{1}{T}-\frac{1}{T_m}\right)\right) + A_2\exp\left(\alpha_2\left(\frac{1}{T}-\frac{1}{T_m}\right)\right) \quad (7)$$

with $T_m = 273$ K. This reveals the following numerical values for the constants used in Eq. (7):

$A_0 = 0.747$

$A_1 = -1.24$

$\alpha_1 = -3.85 \times 10^3$ K

$A_2 = -6.45$

$\alpha_2 = -1.82 \times 10^5$ K.

The derivative of the fitted curve indicates how the apparent activation energy increases towards the melting point. The fitted curve and its derivative are shown in Fig. 10.

Re-analysing the results of Schweizer (1998) shows that his decrease of stiffness corresponds to a slope of $c = -3.6 \times 10^3$ K, equivalent to an apparent activation energy of about 0.3 eV. Considering the fact that those results were obtained at different strain rates with many different samples, the agreement with the present results is considered as good.

The strong decrease of the effective elastic modulus above about -6 °C, and the corresponding increase of the apparent activation energy, is in accordance with the general continuity of state, and hence many studies on ice properties, e.g. Morgan (1991) or Maeno and Nishimura (1978). The measurements of Maeno and Nishimura (1978) on the temperature dependence of the surface conductance of ice show a prominent increase above about -5 °C towards the melting point which is attributed by Petrenko and Whitworth (1999) to the effect of the quasi-liquid layer observed in structural and optical experiments. Similarly, Colbeck (1983) reports a transition between the highly faceted kinetic growth form and the rounded equilibrium form at temperatures above -6 °C; he attributes this observation to the effect of the disordered surface layer. The same pre-melting phenomena are likely responsible for the much easier deformation, the softening, towards the melting point which results in a strongly decreased stiffness of snow. Additional deformation processes as grain boundary sliding might supplement the thermally activated process of dislocation glide. As the strain in our experiments is very small and recoverable the deformation of the snow is likely due to intragranular deformation, i.e. straining of bonds and grains. Intergranular deformation implies grain re-arrangement and typically dominates viscous deformation.

4. Conclusions

By applying the rheometric method as described by Camponovo and Schweizer (2001), the temperature dependence of the effective elastic modulus of snow has been determined. By staying below the proportional elastic limit strain, the modulus describes the truly elastic, recoverable response, and structural changes are avoided so that results from one single sample for a wide range of temperatures (-1 to -20 °C) can be given. This unique approach has for the first time given reliable information on the temperature dependence of one of the most important mechanical properties of snow. The elastic/delayed-elastic modulus is highly relevant for avalanche formation when considering rapid processes such as skier triggering and fracture propagation. The results show that up to about -6 °C the increase is uniform following an Arrhenius relation with an apparent activation energy of about 0.20 eV. Above -6 °C, the effective elastic modulus decreases more rapidly as the melting point is approached, corresponding to pre-melting effects and additional deformation mechanisms. The fact that curves resulting from the different experiments can be shifted so that they coincide shows that structurally similar snow is a rheological simple material. An empirical relation is fitted to the normalized data over the whole temperature range in order to provide a relation for engineering applications. The present study confirms the strong temperature sensitivity of the modulus with the known consequences for the snow-slab release problem as outlined by McClung and Schweizer (1999). Although our measurements were done for a specific type of snow, which however has been fully characterized, the results are likely valid for other snow types, based on the strong coincidence with the properties of ice.

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References

Bader, H., Haefeli, R., Bucher, E., Neher, J., Eckel, O., Thams, Chr., 1939. Der Schnee und seine Metamorphose. Beiträge zur Geologie der Schweiz, Geotechnische Serie, Hydrologie, Lieferung 3. Kümmerly and Frey, Bern, Switzerland (Snow and its metamorphism. U.S. Army Corps of Engineers, Snow, Ice and Permafrost Research Establishment (SIPRE), Translation 14, 1954).

Baunach, T., Fierz, C., Satyawali, P.K., Schneebeli, M., 2001. A model for kinetic grain growth. Ann. Glaciol. 32, 1–6.

Camponovo, C., Schweizer, J., 2001. Rheological measurements of the vicoelastic properties of snow. Ann. Glaciol. 32, 44–50.

Colbeck, S.C., 1983. Ice crystal morphology and growth rates at low supersaturations and high temperatures. J. Appl. Phys. 54 (5), 2677–2682.

Colbeck, S., Akitaya, E., Armstrong, R., Gubler, H., Lafeuille, J., Lied, K., McClung, D.M., Morris, E., 1990. The International Classification for Seasonal Snow on the Ground. International Association of Scientific Hydrology, International Commission on Snow and Ice, Wallingford, Oxon.

de Quervain, M.R., 1966. Problems of avalanche research. International Association of hydrological Sciences Publication 69 (Symposium at Davos 1965—Scientific Aspects of Snow and Ice Avalanches), 1–8.

Findley, W.N., Lai, J.S., Onaran, K., 1976. Creep and Relaxation of Nonlinear Viscoelastic Materials. Dover Publications, New York, USA.

Good, W., 1987. Thin sections, serial cuts and 3-D analysis of snow. International Association of Hydrological Sciences Publication 162 (Symposium at Davos 1986 - Avalanche Formation, Movement and Effects), 35–48.

Goodman, D.J., 1980. Critical stress intensity factor (KIc) measurements at high loading rates for polycrystalline ice. In: Tryde, P. (Ed.), Physics and Mechanics of Ice. Proceedings IUTAM Symposium, Copenhagen, August 6–10, 1979, pp. 129–146.

Hooke, R.LeB., Mellor, M., et al., 1980. Mechancial properties of polycrystalline ice: an assessment of current knowledge and priorities for research. Cold Reg. Sci. Technol. 3, 263–275.

Kirchner, H.O.K., Michot, G., Narita, H., Suzuki, T., 2001. Snow as a foam of ice: plasticity, fracture and the brittle–ductile transition. Philos. Mag. A 81 (9), 2161–2181.

Maeno, N., Nishimura, H., 1978. The electrical properties of ice surfaces. J. Glaciol. 21, 193–205.

McClung, D.M., Schweizer, J., 1999. Skier triggering, snow temperatures and the stability index for dry-slab avalanche initiation. J. Glaciol. 45 (150), 190–200.

Mellor, M., 1975. A review of basic snow mechanics. IAHS Publ. 114, 251–291.

Mellor, M., Testa, R., 1969. Effect of temperature on the creep of ice. J. Glaciol. 8 (52), 131–145.

Morgan, V.I., 1991. High-temperature ice creep tests. Cold Reg. Sci. Technol. 19, 295–300.

Narita, H., 1983. An experimental study on tensile fracture of snow. Contrib. Inst. Low Temp. Sci., Hokkaido Univ. Series A 32, 1–37.

Petrenko, V.F., Whitworth, R.W., 1999. Physics of Ice. Oxford Univ. Press, New York.

Schweizer, J., 1998. Laboratory experiments on shear failure of snow. Ann. Glaciol. 26, 97–102.

Schweizer, J., 1999. Review on dry snow slab avalanche release. Cold Reg. Sci. Technol. 30 (1–3), 43–57.

Shapiro, L.H., Johnson, J.B., Sturm, M., Blaisdell, G.L., 1997. Snow mechanics—review of the state of knowledge and applications. US Army CRREL Report 97-3.

Sinha, N.K., 1978a. Short-term rheology of polycrystalline ice. J. Glaciol. 21 (85), 457–473.

Sinha, N.K., 1978b. Rheology of columnar-grained ice. Exp. Mech. 18 (12), 464–470.

Smith, G.D., Morland, L.W., 1981. Viscous relations of the steady creep of polycrystalline ice. Cold Reg. Sci. Technol. 5, 141–150.

Sturm, M., Benson, C.S., 1997. Vapor transport, grain growth and depth-hoar development in the subarctic snow. J. Glaciol. 43 (143), 42–59.