THE EFFECT OF SLAB AND BED SURFACE STIFFNESS ON THE SKIER-INDUCED SHEAR STRESS IN WEAK SNOWPACK LAYERS

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ABSTRACT: A finite element model was developed to simulate a snowpack with localized surface loading of a skier on a slab of varying stiffness overlaying a thin, weak layer. The goals of this study were (1) to determine the effect of average slab thickness on stress concentrations in the underlying weak layer, and (2) to determine the effect of the stiffness of the slab and the bed surface under the weak layer on stress within the weak layer. The model simulates a snowpack with a slab varying from 0.1 to 0.7 m thick, and with stiffness ratios between the Young's modulus for the slab, weak layer and bed surface varying from 1 to 25. The two-dimensional model assumes snow behaves as a linear-elastic, compressible material, and that a static skier load is applied. The model results for a homogeneous snowpack had peak stresses within 2.5% of the analytical solution for a strip load. The effect of average slab stiffness on the shear stress within the weak layer was assessed by varying the slab stiffness from a soft slab to a stiff slab for various slab thicknesses. The peak shear stress in the weak layer was highest for the softest slab and decreased with increasing slab stiffness. Stress through the snowpack decreased non-linearly with increasing depth.

KEYWORDS: Snow avalanche; avalanche formation; skier triggering; numerical modeling

1. INTRODUCTION

A mountain snowpack during the course of a winter will comprise a layered stratigraphy with different snow properties. Meteorological parameters control how the layers initially form, while a combination of meteorological and mechanical conditions control how the snowpack changes over time (McClung and Schaerer, 1993, p. 48). A typical snowpack may include one or more relatively weak layers which, when overlain by a relatively stiff slab, can act as a failure plane for human-triggered slab avalanches and are the primary cause of avalanche fatalities (Schweizer and Lütschg, 2001; Logan and Atkins, 1996, p. 234).

Föhn (1987) started a trend in snow science toward better understanding how stresses are distributed in a homogeneous snowpack beneath a skier. This work was directed towards an improved understanding of the triggering potential of avalanches due to artificial triggers (e.g. skiers), and ultimately toward improved avalanche prediction. Föhn (1987) recognized that in order for analyses to realistically represent a snowpack, it is essential to consider the effect of layering of the snowpack. The "bridging effect" arises from the variable properties of snow, and is widely understood among professional avalanche workers. Although sometimes a controversial concept, it describes the situation where the upper slab is stiffer than the underlying weak layer so that more skier stress is transmitted laterally in the slab and less is transmitted down through the slab to a buried weak layer. Subtle warming changes in the air temperature can significantly affect the snow stability, allowing stresses to be transmitted to deeper layers in the snowpack and making skier triggering of avalanches more likely (Wilson et al., 1999).

Using a finite element model to simulate a layered snowpack with a buried weak layer, we determine the stress concentration at the interface between the slab and the underlying weak layer. This model is also used to determine the effect of average slab thickness on stress concentration in the weak layer, and compare this with the analytical solution for a homogeneous snowpack.

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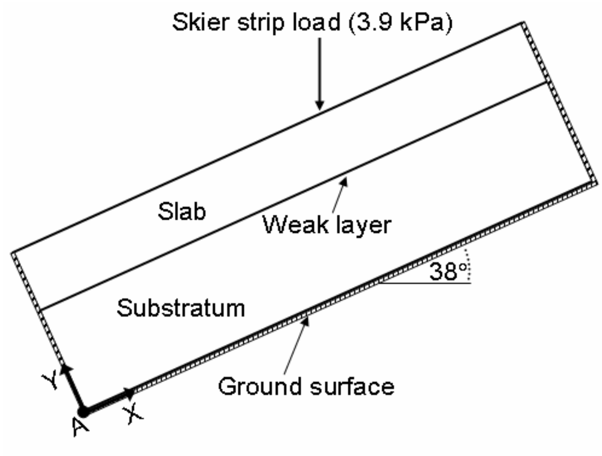

Figure 1. Schematic of finite element model showing layering of the snowpack, X and Y coordinate system, fixed boundaries (shown by the hatched lines), skier strip load of 3.9 kPa, and origin (Point A) used for presentation of results (not to scale).

2. BACKGROUND

Finite element modeling methods were first used to model snow and avalanches in the 1970's and 1980's by Smith et al. (1971), Smith (1972), Curtis and Smith (1974) and Singh (1980). Schweizer (1993) used a finite element model to show that the properties of the slab layering are crucial for skier triggering, and studied the stress distribution in layered snow in response to dynamic loading (Schweizer et al., 1995). Wilson et al. (1999) used finite element models to analyse changes in stress and strain in a weak layer under the load of a skier at different temperatures to determine the effects of warming of the snowpack.

The concept of applying analytical solutions for calculating stresses on a snowpack from a stationary skier was introduced by Föhn (1987), whereby theory was developed to quantify the stability of snow in terms of a stability index, comparable to methods used in soil mechanics. Jamieson (1995) further refined Föhn's stability index for skier triggering that accounted for ski penetration into the snowpack. McClung and Schweizer (1999) discussed in detail the importance of slab temperature and hardness on snow stability and presented a static analysis of the stability index.

In past studies, snow has been considered either as a visco-elastic fluid (e.g. Schweizer, 1993; Schweizer and Camponovo, 2001) or as a linear elastic material (e.g. Smith et al., 1971; Wilson et al., 1999). When snow is subjected to a strain rate greater than approximately 10^{-4} s^{-1}, it deforms linearly (i.e. linear elastic deformation) and then fails as a brittle material (Narita, 1980). Field results that verify the brittle nature of snow are provided by Schweizer and Campanovo (2001) who claim that although "the weighting and loading step of a skier is mainly elastic", a "skier will cause deformation in a potential weak layer that is large and fast enough to start brittle fracture" (Campanovo and Schweizer, 1997). Thus, we simulate the rapid loading rate typical of a skier using a linear elastic model in our analyses.

3. METHODS

Figure 1 shows the conceptual geometry of a simple, layered snowpack in mid-winter. Starting at the bottom of the snowpack at the ground, the substratum consists of older snow that has been changed by meteorological and mechanical conditions to form a relatively stiff base layer. The bottom of this layer is bound by the ground surface, while the top of this layer acts as a bed surface on which snow avalanches release. The weakness on top of the substratum is a thin weak layer. The overlying slab transmits stress from the skier at the surface to the buried weak layer, and can have widely varying thickness and stiffness.

A two-dimensional finite element model was developed to simulate the layered snowpack structure described above. The model was 10 m long and was orientated to represent a snowpack on a 38° slope (Figure 1). This represents the slope angle most commonly associated with skier-triggered avalanches (Logan and Atkins, 1996; Schweizer and Lütschg, 2001; Schweizer and Jamieson, 2001). From top to bottom, the snowpack includes:

1. a slab of variable thickness (0.1 m, 0.3 m, 0.5 m, 0.7 m; measured slope normal);
2. a thin, weak layer of fixed thickness (0.003 m); and
3. an underlying stiff substratum of fixed thickness (2.0 m, measured slope normal)

All dimensions were measured in the x-y plane (i.e. parallel and normal to the slope) as shown in Figure 1, with the origin (Point A) located at the lower left edge of the model. Four model geometries were developed to simulate the four different top slab thicknesses. The model slab thickness values correspond to numbers observed with the most common skier-triggered avalanche incidents, which have a median value of 0.46 m

(Schweizer and Jamieson, 2001), and 89% of which are less than 1.0 m in thickness (Jamieson and Geldsetzer, 1996). Although the thickness of a typical weak layer can range anywhere between 0.005 m and 0.08 m (Jamieson, 1995), failure starts in thin weak layers or at an interface of a thick weak layer (Schweizer et al., 2003). A relatively thin (0.003 m) weak layer was used in this model to simulate a thin failure layer of failure interface. The thickness of the substratum was iteratively chosen to minimize the effect of having a fixed boundary at the bottom of the snowpack, as discussed later in this paper.

The four model geometries use two-dimensional, four-noded quadrilateral (Quad 4) plane strain elements. The smallest model, that for the 0.1 m thick slab, had 357,438 degrees of freedom, while the largest model for the 0.7 m thick slab had 415,558 degrees of freedom. Assumptions used in the analyses included:

1. rapid loading of the snowpack by a skier (e.g. a strain rate greater than 10^{-4} s^{-1}), resulting in a almost linear response to stress;
2. snow within a particular layer was homogeneous and isotropic (Curtis and Smith, 1974);
3. plane strain and plane stress adequately simulate a three-dimensional snowpack (e.g. Smith et al., 1971; Smith, 1972; Föhn, 1987; Schweizer, 1993; Wilson et al., 1999);
4. the top snow surface was free to deform and, except for the skier loading, stress-free (Curtis and Smith, 1974);
5. stress transfer from the snowpack to the ground is considered negligible and thus a fixed lower boundary is assumed; and
6. no relative slippage between adjacent snow layers occurs;
7. ski penetration into the snowpack is negligible.

A 3.9 kPa skier strip load oriented across the slope was applied to the top of the snowpack, 1 m upslope from the middle of the snowpack (i.e. x = 6 m) (Figure 1). The load was applied at this point to minimize the boundary effects from the fixed upslope boundary (i.e. x = 10 m) of the model since the load was concentrated downslope towards the origin to simulate skier loading on a slope. The 3.9 kPa strip load was chosen based on previous analyses by Föhn (1987) and McClung and Schweizer (1999) to effectively model stress near the surface and to minimize the effect of long elements at the surface. This load represents a skier with a weight of 780 N (mass of 80 kg), and assumes that the skier is crossing an extended, uniform slope along a contour line. The skier load is concentrated on a strip 0.2 m wide by 1.0 m long (tips and tails have less effect than the central part of the ski), simulating two wide skis or a snowboard on an angle (McClung and Schweizer, 1999). This problem is solved analytically by McClung and Schweizer (1999) for a homogeneous snowpack using the Boussinesq method, as commonly applied in soil mechanics (e.g. Salm, 1977; Föhn, 1987).

The bottom and side boundaries of the model were assumed to be fixed, while the top surface was left free to deform. Additional analyses were conducted to test the model for boundary effects by removing the confined boundaries at the bottom and sides of the model. The effect of using different element types was evaluated by varying the type of elements and number of nodes.

The assumption that skier penetration into the snowpack is negligible was made to simplify modelling. Although ski penetration is clearly a factor for the stress penetration into the snowpack (e.g. Schweizer et al., 1996), it was not taken into account for either the analytical solution or for the finite element model. This essentially negates the effect of ski penetration when comparing the two solutions. Regardless, errors are introduced into the model with this assumption.

The model material property values (Young's modulus, E, and Poisson's ratio, ν) were chosen to represent typical values from Mellor (1975) and Shapiro et al. (1997) (Table 1). The corresponding hand hardness values were similar to those used by Wilson et al. (1999), ranging from F (fist) to 1F (one finger). Hand hardness is a measurement of resistance of a layer to specific types of gloved hand penetration, and is commonly used by avalanche workers to describe the hardness of layers (Canadian Avalanche Association, 2002, p. 15). The three selected Young's moduli increase

Table 1. Material properties used in the finite element model (Mellor,1975; Shapiro et al., 1997; Wilson et al., 1999)

Snow Condition	Hand hardness	Young's modulus [MPa]	Poisson's ratio	Average density [kg m^{-3}]
Soft	F (fist)	0.3	0.25	105
Medium	4F (4-finger)	1.5	0.25	185
Hard	1F (1-finger)	7.5	0.25	275

by a factor of 5 between values, for a total factor increase of 25 from the soft to hard layers. The density values were based on results provided by Geldsetzer and Jamieson (2001) that relate density to hand hardness.

A Poisson's ratio of 0.49 (nearly perfectly elastic material) was used to verify the homogeneous models with the analytical solutions. Mellor (1975) and Shapiro et al. (1997) showed typical values of Poisson's ratio for snow widely ranging from 0.15 to 0.40. Smith et al. (1971) found that their model results did not vary strongly as a function of Poisson's ratio and that the choice of value was not critical to the model.

Based on these findings, a Poisson's ratio of 0.25 was used for all other materials and runs. The homogeneous verification models were run using reduced integration (i.e. integration only at the centre of the element) to avoid problems associated with using Poisson's ratio values near 0.5 (i.e. overly stiff elements with no volumetric compression). All other model runs used standard integration (i.e. integration at four locations in the elements).

The models were developed using PATRAN (McNeal Schwindler Corp., Costa Mesa, CA, USA) pre-processing software and analysed using ABAQUS (Hibbit, Karlson and Sorrenson, Rhode Island, NY, USA) finite element software. The analyses were performed on a Silicon Graphics Origin 2000 (Silicon Graphics, Calgary, AB, Canada) server at the University of Calgary.

A series of nine material combinations were run in the models for each of the four model geometries (0.1 m, 0.3 m, 0.5 m and 0.7 m), plus verification runs of the homogeneous model using a Poisson's ratio of 0.49 for all four geometries. Thus, a total of 40 model runs were required. Twenty-one trial runs were used to develop and calibrate the initial models, including testing of boundary effects, element size and element type. Table 2 summarizes the array of models runs conducted.

4. RESULTS

4.1 Model Verification

After refinement of the model geometry, mesh and boundary conditions to minimize model errors, the peak stress in the buried weak layer was compared to the analytical solution presented by McClung and Schweizer (1999). This solution is a static analysis that incorporates loading by a skier with a finite-width ski, which in this case was assumed to be 0.2 m. The 0.1 m, 0.3 m, 0.5 m and

Table 2. Summary of finite element model runs showing the ratio of the Young's modulus for adjacent layers. Series of nine material property combinations run for each model slab thickness (0.1, 0.3, 0.5 and 0.7 m)

Run	E_{slab}	E_{wl}	E_{base}[1]	E_{wl} / E_{base}	E_{slab} / E_{wl}
1	0.3	0.3	0.3	1	1
2	1.5	0.3	0.3	1	5
3	7.5	0.3	0.3	1	25
4	0.3	0.3	1.5	0.2	1
5	1.5	0.3	1.5	0.2	5
6	7.5	0.3	1.5	0.2	25
7	0.3	0.3	7.5	0.04	1
8	1.5	0.3	7.5	0.04	5
9	7.5	0.3	7.5	0.04	25

[1] *E_{slab}, E_{wl} and E_{base} represent the Young's modulus for the slab, weak layer and base layer, respectively*

0.7 m homogeneous models had peak stresses within 2.5%, 1.7%, 1.8% and 1.2% of the McClung and Schweizer (1999) strip load analytical solution, respectively. The location of the peak shear stress was within 1 cm of the analytical solution. These results show that the models were in good agreement with the analytical solutions and were considered sufficiently verified. As would be expected, the model error decreases for increasing slab thickness and overall snowpack thickness as the stress and associated errors are distributed through more material and boundary effects become less important.

The development model was compared using both fixed and free end boundary conditions. There was a difference of less than 3% in the peak shear stress in the weak layer when comparing fixed and free boundary conditions. These errors were considered sufficiently small when compared with the variability in material properties, and were consequently ignored. Thus, the fixed end boundary condition was used. The bottom boundary was modeled as a fixed boundary, hence assuming that there is no slippage between the snowpack and the ground surface.

Although the homogeneous model was shown to be sufficiently verified, there is still some uncertainty in the model when the parameters are changed to represent a heterogeneous snowpack. Within the field of soil mechanics, there are methods for determining stresses analytically within layered soil (e.g. Das, 1983) but these methods are empirically derived solutions that are limited to specific types of soil conditions. Additionally, it is unlikely that these methods would be suitable for a snowpack where the slab and weak layers have thicknesses that differ by three orders of magnitude. Given the limited utility of these methods for a layered snowpack, verification of the heterogeneous models was not performed.

4.2 Effect of slab thickness on shear stress in a weak layer

By comparing snow to other materials well studied in fracture mechanics, McClung and Schweizer (1999) suggested that a critical size for fracture propagation during rapid loading is on the order of 0.1 to 1 m. Shorter lengths are considered to be critical during rapid loading by a skier. Thus, rather than assuming a peak stress will result in initiation of a failure in a weak layer, a threshold value of stress must be exceeded for a critical length before a brittle fracture will propagate and initiate an avalanche. Based on these assumptions, we calculate the value of stress that is exceeded for 0.1 m length in the middle row of weak layer elements. The shear stress in the weak layer is also normalized by dividing by the surface stress (3.9 kPa). The results from the nine runs for each of the four slab thicknesses are shown in Table 3 and are plotted in Figure 2.

It can be observed in Table 3 that the shear stress ratio for the 0.7 m slab is reduced approximately 75% when compared to the stress for a 0.1 m slab. The relationship between the shear stress ratio and slab thickness is clearly not linear and is reduced approximately 50% between 0.1 and 0.3 m and the remaining 25% between 0.3 and 0.7 m. The non-linear relationship between stress and depth is comparable to strip-load analytical solution, whereby stress is proportional to the inverse of depth (McClung and Schweizer, 1999).

4.3 Effect of slab stiffness on shear stress in a weak layer

The effect of average slab stiffness on shear stress in the weak layer was assessed by varying the ratio of the slab modulus (E_{slab}) to the weak layer modulus (E_{wl}) for all four slab thicknesses. Ratios of 1, 5 and 25 were chosen to provide a range of slab stiffness from soft (Fist or E_{slab} = 0.3 MPa) to hard (1-Finger or E_{slab} = 7.5 MPa). The results of these analyses are shown in Figures 2a, 2b, 2c and 2d by comparing the three series on each graph for ratios of 1, 5 and 25. It can be observed in all four graphs that the stress ratio is highest for the softest (Fist or E_{slab} = 0.3 MPa) slab, and that stress decreases with increasing slab stiffness (increasing E_{slab}/E_{wl}). Increasing the ratio (E_{slab}/E_{wl}) from 1 to 25 resulted in a decrease in the shear stress ratio of between 22% and 75%, consistent with the concept of

Table 3. Effect of slab layer thickness on shear stress in a weak layer

						Slab thickness (m)			
						0.1	0.3	0.5	0.7
Run	E_{slab}	E_{wl}	E_{base}[2]	E_{wl} / E_{base}	E_{slab} / E_{wl}	Shear stress / surface stress[1]			
1	0.3	0.3	0.3	1	1	0.2714	0.1491	0.0950	0.0686
2	1.5	0.3	0.3	1	5	0.2011	0.0928	0.0568	0.0401
3	7.5	0.3	0.3	1	25	0.1176	0.0279	0.0232	0.0155
4	0.3	0.3	1.5	0.2	1	0.3139	0.2014	0.1134	0.0830
5	1.5	0.3	1.5	0.2	5	0.2693	0.1438	0.0941	0.0687
6	7.5	0.3	1.5	0.2	25	0.1894	0.0878	0.0552	0.0391
7	0.3	0.3	7.5	0.04	1	0.3208	0.2159	0.1173	0.0860
8	1.5	0.3	7.5	0.04	5	0.2961	0.1640	0.1099	0.0812
9	7.5	0.3	7.5	0.04	25	0.2349	0.1258	0.0867	0.0645

[1] Stress that is exceeded for 0.1 m length in middle row of weak layer, divided by the surface stress from a 780 N skier on 1 m long by 0.2 m wide skis/snowboard (3.9 kPa)
[2] E_{slab}, E_{wl} and E_{base} represent the Young's modulus for the slab, weak layer and base layer, respectively

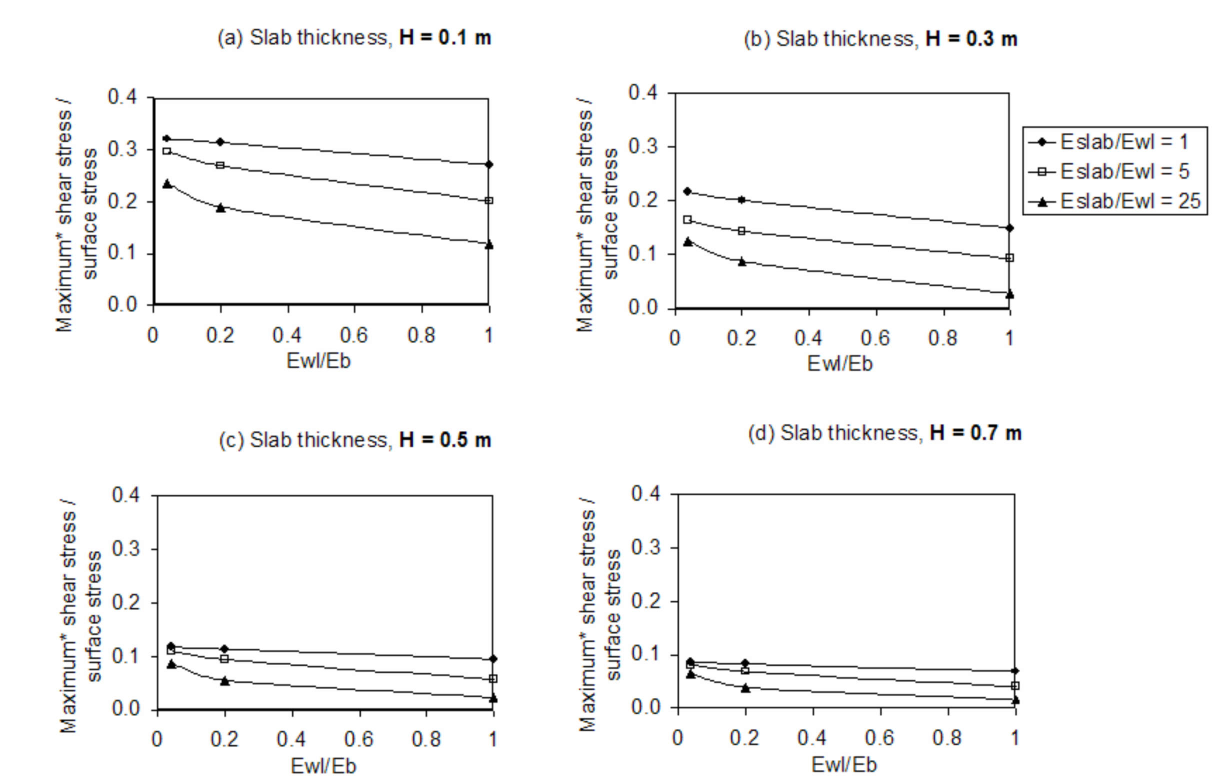

Figure 2. Effect of slab (E_{slab}), weak layer (E_{wl}) and bed surface (E_{base}) modulus on skier-induced shear stress in a weak layer for a slab thickness (a) 0.1 m; (b) 0.3 m; (c) 0.5 m, and; (d) 0.7 m.
** Stress that is exceeded for 10 cm length in middle row of weak layer, divided by the surface stress from an 780 N skier on 1 m long by 0.2 m wide skis/snowboard (3.9 kPa)*

bridging. This variation was also a function of both the slab thickness and the stiffness ratio for the weak layer to the base layer (i.e. E_{wl}/E_{base}). Analogous to this, it can be stated that as the slab softens, the stress below the skier load becomes more concentrated as a given level of stress occurs deeper in the snowpack. This result is in accordance with results from similar studies by Wilson et al. (1999), Schweizer (1993) and Campanovo and Schweizer (1997).

4.4 Effect of bed surface stiffness on shear stress in a weak layer

The third result shown in Figures 2a, 2b, 2c and 2d is the effect of stiffness of the bed surface (E_{base}) on the stress ratio in the weak layer. This is analysed by comparing results along the horizontal axes of the graphs. Although the effect of the stiffness of the bed surface is much less important than the stiffness of the slab, there is a discernable effect on the stress in the weak layer. For example, for a fixed slab thickness of 0.3 m and E_{slab}/E_{wl} of 5, there is an increase in normalized stress from 0.093 for E_{wl}/E_{slab} = 1 to 0.164 for E_{wl}/E_{slab} = 0.04. This is almost a 100 % increase in the normalized stress for a hard (E_{base} = 7.5 MPa) bed surface compared to a soft (E_{base} = 0.3 MPa) bed surface, showing that stiff a bed surface concentrates shear stress in the weak layer located immediately above it. Schweizer and Lütschg (2001) showed that in the Swiss Alps, bed surfaces for skier-triggered slab avalanches often consist of melt-freeze crusts which are harder than the overlying weak layer. Overall, the effect of both slab and bed surface stiffness becomes less important with increasing slab thickness as stress becomes distributed over a wider area. This effect can be observed by comparing Figure 2a (slab thickness 0.1 m) to 2d (slab thickness 0.7 m) and noting how the difference between the three curves decreases (i.e. lessening effect of stiffness of the slab) and the curves become flatter (i.e. lessening effect of stiffness of the bed surface).

5. SUMMARY

A finite element model was used to simulate a snowpack being loaded by a skier. The simulated snowpack was typical of a mountain snowpack, with a base layer of variable stiffness overlain by a thin weak layer in turn overlain by a slab of variable thickness and stiffness.

The shear stress ratio for the 0.7 m slab was approximately 75% less than for a 0.1 m slab. The decreasing effect predicted by the model (line of strip load), has been measured (e.g. Schweizer and Camponovo, 2001) and is reflected by avalanche statistics that show thicker slabs (e.g. thicker than 85 cm) are less often triggered than thin slabs (Schweizer and Jamieson, 2006). This is also consistent with avalanche accident statistics, which show that skier-triggered slab avalanche incidents involve relatively thin slabs with a median value of 0.46 m (Schweizer and Jamieson, 2001). Thus, even though the consequences are higher for triggering a slab of greater thickness (e.g. > 1.0 m), a reduction of the shear stress ratio of approximately 75% between a 0.1 m and 0.7 m slab can reduce the probability of triggering the thicker slab.

When the average slab stiffness was varied, modeling results showed that as a slab softens (i.e. lower Young's modulus), the stress below a skier becomes more concentrated and a given level of stress occurs deeper in the snowpack. This result illustrates the importance of the stiffness of the slab as a result of formation processes (e.g. snowfall density, bonding, temperature), as well as rapid warming of the snowpack which can significantly change the stiffness of a slab in a relatively short time period (e.g. hours). Even if warming effects (and consequent softening of the slab) do not extend as deep as the weak layer, skier-induced shear stress in a weak layer may increase (Wilson et al., 1999). Conversely, for slabs of a given thickness, less shear stress will penetrate through the stiffer slabs, consistent with the practitioners' concept of bridging.

Finally, the effect of the stiffness of the bed surface on the stress ratio in the weak layer was analysed. Although the effect of the stiffness of the bed surface was less than that of the stiffness of the slab, the effect was discernable and is also considered important. This result demonstrates the importance of recognizing relatively stiff bed surfaces when conducting snow profiles, especially melt-freeze crusts that may be present immediately beneath a weak layer. These layers may concentrate stress in the overlying weak layer, and contribute to the likelihood of triggering an avalanche of the overlying slab.

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